OT REPORT 75-76

A VERSATILE THREE-DIMENSIONAL RAY TRACING COMPUTER PROGRAM FOR RADIO WAVES IN THE IONOSPHERE

OT

U.S. DEPARTMENT OF COMMERCE Office of Telecommunications

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OT REPORT 75-76

A VERSATILE THREE-DIMENSIONAL RAY TRACING COMPUTER PROGRAM
FOR RADIO WAVES IN THE IONOSPHERE

by R. Michael Jones and Judith J. Stephenson

The 8th line from the bottom of page 9 should read:
(i) the dispersion relation cannot be exactly satisfied, or

The first line in the 3rd complete paragraph on page 12 should read:
Similarly, the AHWFNC (Appleton-Hartree, with field, no colli-

Line BACK052 in SUBROUTINE BACK UP on page 75 should be replaced by:

```
      H=R(1)-EARTHR                                                    BACK0519
      STEP=(-R(4)-SQRT(R(4)**2-2.*(H-HS)*R(4)/DRDT(1)*DRDT(4)))/DRDT(4) BACK052
```

Line BACK060 in SUBROUTINE BACK UP on page 75 should read:

```
      IF(ABS(R(1)-EARTHR-HS).LT..5E-4.AND.ABS(STEP).LT.1.) GO TO 50     BACK060
```

Line PRIN008 in SUBROUTINE PRINTR on page 78 should read:

```
     1      LPOLAR(2),SGN                                               PRIN008
```

Line PRIN125 in SUBROUTINE PRINTR on page 80 should read:

```
      RANGE=EARTHR*ATAN2(RCE,EARTHR+EPS+XMTRH)                          PRIN125
```

Following line WFWC006 in SUBROUTINE AHWFWC on page 93, insert the line:

```
     2       ,SGN                                                       WFWC0065
```

Line BQNC020 in SUBROUTINE BQWFNC on page 100 should read:

```
      REAL N2,NNP,LPOLAR,LPOLRI,KR,KTH,KPH,K2,KDOTY,K4,KDOTY2,          BQNC020
```

Line TABX064 in SUBROUTINE TABLEX on page 112 should read:

```
      PXPR=PXPTH=PXPPH=0.                                               TABX064
```

Following line CHAP024 in SUBROUTINE CHAPX on page 116, insert the line:

```
      PXPPH=0.                                                          CHAP0245
```

Line VCHA010 in SUBROUTINE VCHAPX on page 117 should read:

```
      X=PXPR=PXPTH=PXPPH=0.                                          VCHA010
```

Line DCHA013 in SUBROUTINE DCHAPT on page 119 should be replaced by:

```
      HMAX1=HM1+EARTHE*THETA2                                        DCHA013
      HMAX2=HM2+EARTHE*THETA2                                        DCHA0131
      HMAX=HMAX1                                                     DCHA0132
      IF(FC2.GT.FC1.OR.FC2.EQ.FC1.AND.HMAX2.GT.HMAX1) HMAX=HMAX2     DCHA0133
```

Line DCHA014 in SUBROUTINE DCHAPT on page 119 should read:

```
      X=PXPR=PXPTH=PXPPH=0.                                          DCHA014
```

Line DCHA017 in SUBROUTINE DCHAPT on page 119 should read:

```
      Z1=(H-HMAX1)/SH1                                               DCHA017
```

Line LINE013 in SUBROUTINE LINEAR on page 120 should read:

```
      X=PXPR=PXPTH=PXPPH=0.                                          LINE013
```

Line PARA012 in SUBROUTINE QPARAB on page 121 should read:

```
      X=PXPR=PXPTH=PXPPH=0.                                          PARA012
```

Following line BULG038 in SUBROUTINE BULGE on page 123, insert the line:

```
      PXPPH=0.                                                       BULG0385
```

Following line EXPX014 in SUBROUTINE EXPX on page 124, insert the line:

```
      PXPTH=PXPPH=0.                                                 EXPX0145
```

The equation for the gyrofrequency near the top of page 143 should read:

$$F_H = F_{H_0} (R_0/(R_0 + h))^3 (1 + 3 \cos^2\theta)^{1/2}$$

where θ is the geomagnetic colatitude.

Line TABZ015 in SUBROUTINE TABLEZ on page 153 should read:

```
      IF (READNU.EQ.0.) GO TO 10                                     TABZ015
```

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OFFICE OF TELECOMMUNICATIONS
John M. Richardson, Acting Director

October 1975

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PREFACE

This report documents the latest version of the three-dimensional ray tracing program originally described in "A Three-Dimensional Ray Tracing Computer Program," by R. M. Jones, ESSA Technical Report IER 17-ITSA 17, and later modified in "Modifications to the Three-Dimensional Ray Tracing Program Described in IER 17-ITSA 17," by R. M. Jones, ESSA Technical Memorandum ERLTM-ITS 134. This report replaces all of the material contained in the above two reports.

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A VERSATILE THREE-DIMENSIONAL RAY TRACING COMPUTER PROGRAM FOR RADIO WAVES IN THE IONOSPHERE

R. Michael Jones* and Judith J. Stephenson**

This report describes an accurate, versatile FORTRAN computer program for tracing rays through an anisotropic medium whose index of refraction varies continuously in three dimensions. Although developed to calculate the propagation of radio waves in the ionosphere, the program can be easily modified to do other types of ray tracing because of its organization into subroutines.

The program can represent the refractive index by either the Appleton-Hartree or the Sen-Wyller formula, and has several ionospheric models for electron density, perturbations to the electron density (irregularities), the earth's magnetic field, and electron collision frequency.

For each path, the program can calculate group path length, phase path length, absorption, Doppler shift due to a time-varying ionosphere, and geometrical path length. In addition to printing these parameters and the direction of the wave normal at various points along the ray path, the program can plot the projection of the ray path on any vertical plane or on the ground and punch the main characteristics of each ray path on cards.

The documentation includes equations, flow charts, program listings with comments, definitions of program variables, deck set-ups, descriptions of input and output, and a sample case.

Key words: Ray tracing, computer program, radio waves, ionosphere, three-dimensional, Appleton-Hartree formula, Sen-Wyller formula.

---

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1. INTRODUCTION

This report describes a three-dimensional ray tracing program written in FORTRAN language for the CDC-3800 computer. Copies of the program deck are available from the Institute for Telecommunication Sciences.

Earlier versions of this program have been in use now for over nine years, both by us and by people scattered all over the world. During that time we have improved and modified the program to the extent that we now need to document these changes so that the present program will be easier to use. We have included the input parameter forms that we use to request ray path calculations because they give nearly all the necessary input data and describe the electron density, collision frequency, and magnetic field models.

2. GENERAL DESCRIPTION

This computer program traces the path of radio wave through a user-specified model of the ionosphere when given the transmitter location (longitude, latitude, and height above the ground), the frequency of the wave, the direction of transmission (both elevation and azimuth), the receiver height, and the maximum number of hops wanted.

3. RAY TRACING EQUATIONS

The program calculates ray paths by numerically integrating Hamilton's equations. Lighthill (1965) gives Hamilton's equations in four dimensions (three spatial and one time) for Cartesian coordinates. Haselgrove (1954) gives Hamilton's equations in three dimensions for spherical polar coordinates. Combining the two gives Hamilton's equations in four dimensions in which the three spatial coordinates are spherical polar (see Table 1 for a definition of the symbols):

$$\frac{dr}{d\tau} = \frac{\partial H}{\partial k_r}, \qquad (1)$$

Table 1. List of the More Important Symbols

A	In section 3, absorption in decibels.
B_o	Magnetic induction of earth's magnetic field.
c	Speed of electromagnetic waves in free space.
C	Cosine of the angle of incidence on the ionosphere.
e	Charge of the electron (a negative number).
$F(w)$	$F(w) = w C_{3/2}(w) + i \frac{5}{2} C_{5/2}(w) = \frac{1}{3/2!} \int_o^\infty \frac{t^{3/2} \exp(-t) dt}{w - it}$. (Davies, 1965, p. 86)
f	Wave frequency.
Δf	Frequency shift of a wave due to a time varying ionosphere (sometimes called Doppler shift).
f_H	Gyro frequency for electrons, $\lvert e \rvert B_o / 2\pi m$.
f_N	Plasma frequency, $(Ne^2/4\pi^2 \epsilon_o m)^{\frac{1}{2}}$.
$G(w)$	$wF(w)$.
H	Hamiltonian.
k_r, k_θ, k_φ	Components of the propagation vector in the r, θ, φ directions--a vector perpendicular to the wave front having a magnitude $2\pi/\lambda = \omega/v$.
m	Mass of electron.
N	Number of electrons per unit volume.
n	Phase refractive index (in general complex).
n'	Group refractive index (in general complex).
P	Phase path length, phase of wave divided by free space wave number $2\pi/\lambda_o$.
P'	Group path length ct.
r, θ, φ	Coordinates of a point in spherical polar coordinates.
s	Geometric ray path length.
S	Sine of the angle of incidence on the ionosphere.

Table 1. (Continued)

t	Time, travel time of a wave packet.
U	$1 - iZ$ in the Appleton-Hartree formula or $Z/F(1/Z) = 1/G(1/Z)$ in the Sen-Wyller formula.
V_r, V_θ, V_φ	Components of the wave normal direction in the r, θ, and φ directions, normalized so that $V_r^2 + V_\theta^2 + V_\varphi^2 = \text{Real}\{n^2\}$.
v	Phase velocity.
X	$\omega_N^2/\omega^2 = f_N^2/f^2 = Ne^2/(\epsilon_o m\omega^2)$.
Y	$\omega_H/\omega = f_H/f$.
Y_L	$Y \cos \psi$.
Y_T	$Y \sin \psi$.
Z	υ/ω or υ_m/ω.
ϵ_o	Electric permittivity of free space.
θ	Colatitude in spherical polar coordinates.
λ	Wavelength.
λ_o	Wavelength in free space.
υ	Electron collision frequency.
υ_m	Mean electron collision frequency.
ρ	Characteristic wave polarization (definition in Table 6).
ρ_L	Longitudinal polarization (definition in Table 6).
τ	Independent variable in Hamilton's equations.
φ	Longitude in spherical polar coordinates.
ψ	Angle between wave normal and $-B_o$.
ω	$2\pi f$, angular wave frequency.
$\Delta\omega$	$2\pi\Delta f$, angular frequency shift.
ω_H	$2\pi f_H = \lvert e \rvert B_o/m$, angular gyrofrequency.
ω_N	$2\pi f_N = (Ne^2/\epsilon_o m)^{\frac{1}{2}}$, angular plasma frequency.

$$\frac{d\theta}{d\tau} = \frac{1}{r}\frac{\partial H}{\partial k_\theta} \quad , \tag{2}$$

$$\frac{d\varphi}{d\tau} = \frac{1}{r\sin\theta}\frac{\partial H}{\partial k_\varphi} \quad , \tag{3}$$

$$\frac{dt}{d\tau} = -\frac{\partial H}{\partial \omega} \quad , \tag{4}$$

$$\frac{dk_r}{d\tau} = -\frac{\partial H}{\partial r} + k_\theta \frac{d\theta}{d\tau} + k_\varphi \sin\theta \frac{d\varphi}{d\tau} \quad , \tag{5}$$

$$\frac{dk_\theta}{d\tau} = \frac{1}{r}\left(-\frac{\partial H}{\partial \theta} - k_\theta \frac{dr}{d\tau} + k_\varphi\, r\cos\theta \frac{d\varphi}{d\tau}\right) \quad , \tag{6}$$

$$\frac{dk_\varphi}{d\tau} = \frac{1}{r\sin\theta}\left(-\frac{\partial H}{\partial \varphi} - k_\varphi \sin\theta \frac{dr}{d\tau} - k_\varphi r\cos\theta \frac{d\theta}{d\tau}\right) \quad , \tag{7}$$

$$\frac{d\omega}{d\tau} = \frac{\partial H}{\partial t} \quad , \tag{8}$$

The variables r, θ, φ are the spherical polar coordinates of a point on the ray path; k_r, k_θ, and k_φ are the components of the propagation vector (wave normal direction normalized so that in free space

$$k_r^2 + k_\theta^2 + k_\varphi^2 = \frac{\omega^2}{c^2} \quad , \tag{9}$$

where $\omega = 2\pi f$ is the angular frequency of the wave and c is the speed of propagation of electromagnetic waves in free space); t is time, in (4) it is the propagation time of a wave packet, in (8) it expresses the variation with time of a time varying medium; τ is a parameter whose value depends on the choice of the Hamiltonian H.

For actual calculation, the ray tracing program uses group path $P' = ct$ as the independent variable because the derivatives with respect to P' are independent of the choice of Hamiltonian, allowing the program to switch Hamiltonians in the middle of a path. This choice automatically causes the program to take smaller steps in real path length near reflection where the calculations are more critical. The resulting equations obtained by dividing (1) through (8) by c times (4) are:

$$\frac{dr}{dP'} = -\frac{1}{c}\frac{\partial H/\partial k_r}{\partial H/\partial \omega} , \tag{9}$$

$$\frac{d\theta}{dP'} = -\frac{1}{rc}\frac{\partial H/\partial k_\theta}{\partial H/\partial \omega} , \tag{10}$$

$$\frac{d\varphi}{dP'} = -\frac{1}{rc\,\sin\theta}\frac{\partial H/\partial k_\varphi}{\partial H/\partial \omega} , \tag{11}$$

$$\frac{dk_r}{dP'} = \frac{1}{c}\frac{\partial H/\partial r}{\partial H/\partial \omega} + k_\theta\frac{d\theta}{dP'} + k_\varphi \sin\theta\frac{d\varphi}{dP'} , \tag{12}$$

$$\frac{dk_\theta}{dP'} = \frac{1}{r}\left(\frac{1}{c}\frac{\partial H/\partial \theta}{\partial H/\partial \omega} - k_\theta\frac{dr}{dP'} + k_\varphi\, r\cos\theta\frac{d\varphi}{dP'}\right) , \tag{13}$$

$$\frac{dk_\varphi}{dP'} = \frac{1}{r\sin\theta}\left(\frac{1}{c}\frac{\partial H/\partial \varphi}{\partial H/\partial \omega} - k_\varphi \sin\theta\frac{dr}{dP'} - k_\varphi\, r\cos\theta\frac{d\theta}{dP'}\right) , \tag{14}$$

$$\frac{d(\Delta f)}{dP'} = \frac{1}{2\pi}\frac{d\Delta\omega}{dP'} = \frac{1}{2\pi}\frac{d\omega}{dP'} \tag{15}$$

$$= -\frac{1}{2\pi}\frac{\partial H/\partial t}{\partial H/\partial \omega} .$$

Equation (15) for the frequency shift of a wave propagating through a time varying medium follows directly from Hamilton's equations (4) and (8). An alternative derivation is given by Bennett (1967). For large frequency shifts, the frequency shift should be accumulated along the ray path and the shifted frequency used in calculations at each point on the ray path. Equations (1) through (8) imply that all eight dependent variables vary along the path, and that at each point on the path the instantaneous value of all parameters (including frequency) is used in further evaluations of the equations. However, the time variation of the ionosphere due to natural causes (such as solar flares) is so slow that the resulting frequency shifts are small enough (less than one part in 10^5) to have negligible effect on the propagation. For this reason, the program calculates frequency shift to compare with frequency shift measurements, but does not adjust the carrier frequency of the wave used in the propagation calculations.

The first six differential equations (9) through (14) are always integrated. The user can choose whether to have the program integrate (15) to calculate the frequency shift.

There are three other quantities that can be calculated by integration along the ray path. The phase path P (phase divided by the free space wavenumber $2\pi/\lambda_o = \omega/c$) is calculated by integrating

$$\frac{dP}{dP'} = \frac{c}{\omega}\left(k_r \frac{dr}{dP'} + k_\theta\, r \frac{d\theta}{dP'} + k_\varphi\, r \sin\theta \frac{d\varphi}{dP'}\right)$$

$$= -\frac{1}{\omega}\,\frac{k_r \dfrac{\partial H}{\partial k_r} + k_\theta \dfrac{\partial H}{\partial k_\theta} + k_\varphi \dfrac{\partial H}{\partial k_\varphi}}{\partial H/\partial \omega} \quad . \qquad (16)$$

If the absorption per wavelength is small (as it must be for this type of ray tracing to be valid), then an approximate formula can be integrated

to give the absorption in decibels

$$\frac{dA}{dP'} = -\frac{10}{\log_e 10}\,\frac{\omega}{c}\,\frac{\mathrm{imag}\left(\frac{\omega^2}{c^2}\,n^2\right)}{k_r^2 + k_\theta^2 + k_\varphi^2}\,\frac{dP}{dP'}$$

$$= \frac{10}{\log_e 10}\,\frac{\mathrm{imag}\left(\frac{\omega^2}{c^2}\,n^2\right)}{k_r^2 + k_\theta^2 + k_\varphi^2}\,\frac{k_r\frac{\partial H}{\partial k_r} + k_\theta\frac{\partial H}{\partial k_\theta} + k_\varphi\frac{\partial H}{\partial k_\varphi}}{c\;\partial H/\partial\omega}\,, \qquad (17)$$

where n is the (complex) phase refractive index. The geometrical path length of the ray can be calculated by integrating

$$\frac{ds}{dP'} = \sqrt{\left(\frac{dr}{dP'}\right)^2 + r^2\left(\frac{d\theta}{dP'}\right)^2 + r^2\sin^2\theta\left(\frac{d\varphi}{dP'}\right)^2}$$

$$= -\frac{\sqrt{\left(\frac{\partial H}{\partial k_r}\right)^2 + \left(\frac{\partial H}{\partial k_\theta}\right)^2 + \left(\frac{\partial H}{\partial k_\varphi}\right)^2}}{c\;\partial H/\partial\omega}\,. \qquad (18)$$

The user can choose to have frequency shift, phase path, absorption, or path length calculated using equations (15), (16), (17), or (18) and printed by setting the appropriate value in the input W array. (W59, W57, W58, W60 in Table 2.)

If the user wants to add differential equations to the program, he can do so by modifying subroutine HAMLTN, which evaluates Hamilton's equations.

The Hamiltonian and its derivatives are calculated by one of the versions of subroutine RINDEX, which also calculates the phase refractive index and its derivatives.

4. CHOOSING AND CALCULATING THE HAMILTONIAN

Because Hamilton's equations guarantee that the Hamiltonian is constant along the ray path and because it is desirable to have the dispersion

relation satisfied at each point on the ray path, it is usual to write the dispersion relation in the form H = constant and choose that H as the Hamiltonian. Two problems arise. First, in a lossy medium the dispersion relation is complex, so that the resulting complex Hamiltonian gives ray paths having complex coordinates when used in Hamilton's equations. Second, in some cases some forms of the dispersion relation have computational advantages over others when used as a Hamiltonian.

Allowing the coordinates of the ray path to assume complex values is called ray tracing in complex space (Budden and Jull, 1964; Jones, 1970; Budden and Terry, 1971) which is the extension to three dimensions of the phase integral method (Budden, 1961). Ray tracing in complex space is necessary to calculate the propagation of LF radio waves in the D region of the ionosphere (Jones, 1970), and it may also be needed for some medium frequencies.

However, the effect of losses on the ray path of HF radio waves in the ionosphere is probably small, so that the only effect of losses is to attenuate the signal. For this case, then, it is desirable to find a prescription for calculating ray paths having real coordinates. Several methods exist for doing this, and except for computational difficulties, one is probably as good as another. One should recognize that along the ray path:

(1) the dispersion relation cannot be exactly satisfied, or

(2) Hamilton's equations cannot be satisfied, or

(3) both of the above.

In our program, we have chosen to keep Hamilton's equations and require only the real part of the dispersion relation to be satisfied, neglecting the imaginary part. Another approach (Suchy, 1972) is to alter Hamilton's equations so that the full complex dispersion relation is still satisfied along a ray path having real coordinates. We are

reasonably certain that for any situation in which Suchy's method gives significantly different answers from ours, neither method is valid; ray tracing in complex space or an equivalent method would then be required.

Three choices for the Hamiltonian illustrate the computational difficulties involved. Haselgrove (1954) used the following Hamiltonian

$$H = \frac{c}{\omega} \frac{(k_r^2 + k_\theta^2 + k_\varphi^2)^{\frac{1}{2}}}{\text{real}(n)} - 1 , \tag{19}$$

which, except for the effects of errors in the numerical integration and the value of the independent variable, is equivalent to

$$H = 1 - \frac{\omega}{c} \frac{\text{real}(n)}{(k_r^2 + k_\theta^2 + k_\varphi^2)^{\frac{1}{2}}}$$

$$= \text{real} \left\{ 1 - \frac{\omega}{c} \frac{n}{(k_r^2 + k_\theta^2 + k_\varphi^2)^{\frac{1}{2}}} \right\} . \tag{20}$$

There are eight versions of the subroutine RINDEX which calculate the Hamiltonian and its partial derivatives. (Eight versions allow the user to choose the Appleton-Hartree formula or the Sen-Wyller formula, and to include or ignore the earth's magnetic field and collisions.) Six of these versions (subroutines AHWFWC, AHWFNC, AHNFWC, AHNFNC, SWWF, and SWNF) use the following Hamiltonian:

$$H = \frac{1}{2} \left(\frac{c^2}{\omega^2} (k_r^2 + k_\theta^2 + k_\varphi^2) - \text{real}(n^2) \right)$$

$$= \text{real} \left\{ \frac{1}{2} \left(\frac{c^2}{\omega^2} (k_r^2 + k_\theta^2 + k_\varphi^2) - n^2 \right) \right\} . \tag{21}$$

The other two versions (subroutines BQWFWC and BQWFNC) use as a Hamiltonian the real part of the quadratic equation whose solution is the Appleton-Hartree formula (Budden, 1961)

$$H = \text{real}\left\{[(U-X)\,U^2 - Y^2U]\,c^4k^4 + X(k\cdot Y)^2\,c^4k^2 + \right.$$

$$+ [-2U(U-X)^2 + Y^2(2U-X)]\,c^2k^2\omega^2 - X(k\cdot Y)^2\,c^2\omega^2 +$$

$$\left. + [(U-X)^2 - Y^2]\,(U-X)\,\omega^4\right\} \qquad (22)$$

except in or near free space (defined by $X < 0.1$) where they also use (21) as the Hamiltonian. In (22), $U = 1 - iZ$, and X, Y, and Z are the usual magnetoionic parameters.

In a lossy medium, the Hamiltonians in (20), (21), and (22) determine slightly different ray paths, but the differences are significant only when it is no longer valid to represent ray paths with coordinates that are real rather than complex. In fact, this is a weak criterion. The ray paths determined by these three Hamiltonians will become invalid before there are noticeable differences between the three ray paths. In a lossless medium, the above three Hamiltonians determine identical ray paths (except for integration errors).

For either a lossy or lossless medium, some of the above three Hamiltonians have computational difficulties. Special care must be taken in using (19) or (20) in an evanescent region (which is frequently necessary at or near vertical incidence because the numerical integration subroutine usually requires the evaluation of the differential equations not only on the ray path, but also at points near the ray path). For instance, in a lossless medium, real (n) is zero in an evanescent region, which leads to problems in (19) and (20). This problem will not arise in (21) because real (n^2) is well behaved in or at the boundary of an evanescent region, nor will it occur in using (22).

Neither (20) nor (21) (nor any other Hamiltonian based on the refractive index) will work for a ray passing through a spitze (Davies, 1965, p. 202) because the refractive index is indeterminate at a spitze, and

some of the derivatives of n diverge. So far, we have had no problems using (22) to calculate ray paths through a spitze with or without collisions.

However, the Hamiltonian in (22) will not work in or near free space because all of its derivatives are zero in free space. This problem is related to (22) not being able to distinguish between ordinary and extraordinary waves. To get started, the program uses (21) until the electron density is large enough that X is equal or greater than 1/10.

As far as we can tell, the AHWFWC (Appleton-Hartree, with field, with collisions) version of subroutine RINDEX has been made obsolete by the BQWFWC (Booker quartic, with field, with collisions) version. The latter will do everything the AHWFWC version will do and in addition it will calculate rays through spitzes. A few trial runs, however, indicate that AHWFWC runs about 30 percent faster than BQWFWC.

Similarly, the AHWFNC (Appleton-Hartree, with field, no collisions) version has been made obsolete by the BQWFNC version, which apparently runs just as fast as the AHWFNC version. We are continuing to include the AHWFNC version just in case there are undiscovered problems with the BQWFNC version.

In addition to the Appleton-Hartree formula, which is based on a constant collision frequency, the program also includes the generalized formula of Sen and Wyller (1960), which assumes a Maxwell-Boltzman distribution of electron energy and a collision frequency proportional to energy. Two versions of subroutine RINDEX use the Sen-Wyller formula for calculating the refractive index and the resulting Hamiltonian with its derivatives. These are SWWF, which includes the effects of the earth's magnetic field, and SWNF, which neglects the Earth's magnetic field. The SWWF version will probably not work for calculating rays through a spitze. It would be possible to make a version which used as

its Hamiltonian the quadratic equation whose solution is the Sen-Wyller formula for calculating rays through a spitze, but it is unlikely that we will ever do that.

The versions of subroutine RINDEX that use (21) for a Hamiltonian use the following formulas for calculating the derivatives of that Hamiltonian.

$$\frac{\partial H}{\partial t} = -n \frac{\partial n}{\partial t} \quad , \tag{23}$$

$$\frac{\partial H}{\partial r} = -n \frac{\partial n}{\partial r} \quad , \tag{24}$$

$$\frac{\partial H}{\partial \theta} = -n \frac{\partial n}{\partial \theta} \quad , \tag{25}$$

$$\frac{\partial H}{\partial \varphi} = -n \frac{\partial n}{\partial \varphi} \quad , \tag{26}$$

$$\frac{\partial H}{\partial \omega} = -\frac{n\, n'}{\omega} \quad , \tag{27}$$

$$\frac{\partial H}{\partial k_r} = \frac{c^2}{\omega^2} k_r - \frac{c}{\omega} n \frac{\partial n}{\partial V_r} \quad , \tag{28}$$

$$\frac{\partial H}{\partial k_\theta} = \frac{c^2}{\omega^2} k_\theta - \frac{c}{\omega} n \frac{\partial n}{\partial V_\theta} \quad , \tag{29}$$

$$\frac{\partial H}{\partial k_\varphi} = \frac{c^2}{\omega^2} k_\varphi - \frac{c}{\omega} n \frac{\partial n}{\partial V_\varphi} \quad , \tag{30}$$

$$\vec{k} \cdot \frac{\partial H}{\partial \vec{k}} = k_r \frac{\partial H}{\partial k_r} + k_\theta \frac{\partial H}{\partial k_\theta} + k_\varphi \frac{\partial H}{\partial k_\varphi} = n^2 \quad , \tag{31}$$

where n' is the group refractive index defined by

$$n' = n + f\frac{dn}{df} = n + \omega\frac{dn}{d\omega} \quad , \tag{32}$$

and V_r, V_θ, and V_φ are the components of the wave normal direction in the r, θ, and φ directions normalized so that

$$V_r^2 + V_\theta^2 + V_\varphi^2 = \text{Real}\,\{n^2\} \quad . \tag{33}$$

The derivatives of the Hamiltonian in (22) are given in section 5. 5.

5. REFRACTIVE INDEX EQUATIONS

The refractive index equations used in this ray tracing program are based either on the Appleton-Hartree formula (Budden, 1961) or on the generalized formula of Sen and Wyller (1960). There are eight versions of SUBROUTINE RINDEX, the subroutine that calculates the refractive index and its gradient:

(1) Appleton-Hartree formula with field, with collisions.

(2) Appleton-Hartree formula with field, no collisions.

(3) Appleton-Hartree formula with collisions, no field.

(4) Appleton-Hartree formula no field, no collisions.

(5) Booker quartic with field, with collisions.

(6) Booker quartic with field, no collisions.

(7) Sen-Wyller formula with field.

(8) Sen-Wyller formula, no field.

Each of these eight versions calculates n^2, nn', $n\,\partial n/\partial r$, $n\,\partial n/\partial\theta$, $n\,\partial n/\partial\varphi$, $n\,\partial n/\partial V_r$, $n\,\partial n/\partial V_\theta$, $n\,\partial n/\partial V_\varphi$, $n\,\partial n/\partial t$, and the polarization, where n is the complex phase refractive index; n' is the complex group refractive index; r, θ, and φ are the spherical polar coordinates of a point on the ray path, and V_r, V_θ, and V_φ are the components of the wave normal direction in the r, θ, and φ directions. The quantities

X, $\partial X/\partial r$, $\partial X/\partial \theta$, $\partial X/\partial \varphi$, and $\partial X/\partial t$ are supplied by one of the versions of subroutine ELECTX which defines the electron density model. The quantities Y, $\partial Y/\partial r$, $\partial Y/\partial \theta$, $\partial Y/\partial \varphi$, Y_r, $\partial Y_r/\partial r$, $\partial Y_r/\partial \theta$, $\partial Y_r/\partial \varphi$, Y_θ, $\partial Y_\theta/\partial r$, $\partial Y_\theta/\partial \theta$, $\partial Y_\varphi/\partial \varphi$, Y_φ, $\partial Y_\varphi/\partial r$, $\partial Y_\varphi/\partial \theta$, and $\partial Y_\varphi/\partial \varphi$ are supplied by one of the versions of subroutine MAGY which defines the magnetic field model. The quantities Z, $\partial Z/\partial r$, $\partial Z/\partial \theta$, and $\partial Z/\partial \varphi$ are supplied by one of the versions of subroutine COLFRZ which defines the collision frequency model.

In our formulation, we have tried to avoid using multivalued functions, such as the square root or $\cos^{-1}$, wherever possible. Only twice do we use the square root. One instance is the square root in the Appleton-Hartree formula, unavoidable without adding more differential equations to the system. The second instance is a square root used to calculate polarization. This latter use is unimportant because the polarization is not used in the ray tracing equations.

It is desirable to avoid multivalued functions because, unless extreme care is used, the value of such a function can change discontinuously from one point on the ray path to the next. A particularly troublesome case occurs at reflection for vertical incidence. At that point, the real part of n^2 goes through zero, and n changes from approximately purely real to approximately purely imaginary. Since the numerical integration subroutine usually requires the evaluation of the differential equations not only on the ray path, but also at points near the ray path, it is necessary to be able to evaluate the differential equations above the reflection height, that is, in an evanescent region.

We have found that it is possible to regroup the variables in the equations to avoid this problem: we calculate the real part of n^2 and its derivatives instead of the real part of n and its derivatives. And we calculate nn' instead of n'.

It was not easy to avoid using multivalued functions, however. Many of the usual parameters used to compute the refractive index require the use of multivalued functions in their calculation. Thus, we couldn't calculate

$$V = \sqrt{V_r^2 + V_\theta^2 + V_\varphi^2}$$

nor cos ψ, nor sin ψ, where ψ is the angle between the wave normal direction and the earth's magnetic field. Thus, we also could not calculate

$$Y_L = Y \cos \psi$$

nor

$$Y_T = Y \sin \psi .$$

The most difficult part of avoiding the use of multivalued functions was in calculating the derivatives.

The following is a list of the equations calculated by the eight versions of subroutine RINDEX.

5.1 Appleton-Hartree Formula with Field, with Collisions

The square of the complex phase refractive index is given by

$$n^2 = 1-2X \frac{1-iZ-X}{2(1-iZ)(1-iZ-X) - Y_T^2 \pm \sqrt{Y_T^4 + 4Y_L^2(1-iZ-X)^2}} , \tag{34}$$

where

$$X = \frac{f_N^2}{f^2} , \tag{35}$$

$$Y = \frac{f_H}{f} , \tag{36}$$

$$Z = \frac{\nu}{2\pi f} , \tag{37}$$

$$Y_T = Y \sin \psi, \tag{38}$$

$$Y_L = Y \cos \psi, \tag{39}$$

f_N is the plasma frequency, f_H is the electron gyrofrequency, ν is the electron collision frequency, f is the wave frequency, and ψ is the angle between the wave normal direction and the earth's magnetic field.

The following equations parallel the formulas in this version of RINDEX.

$$V^2 = V_r^2 + V_\theta^2 + V_\varphi^2 \tag{40}$$

$$V \cdot Y = V_r Y_r + V_\theta Y_\theta + V_\varphi Y_\varphi \tag{41}$$

$$\frac{Y_L}{V} = \frac{V \cdot Y}{V^2} \tag{42}$$

$$Y_L^2 = \frac{(V \cdot Y)^2}{V^2} \tag{43}$$

$$Y_T^2 = Y^2 - Y_L^2 \tag{44}$$

$$Y_T^4 = \left(Y_T^2\right)^2 \tag{45}$$

$$U = 1 - iZ \tag{46}$$

$$RAD = \pm\sqrt{Y_T^4 + 4Y_L^2\ (U-X)^2} \tag{47}$$

$$D = 2U(U-X) - Y_T^2 + RAD \tag{48}$$

$$n^2 = 1 - \frac{2X(U-X)}{D} \tag{49}$$

$$\frac{n}{Y_L Y_T}\ \frac{\partial n}{\partial \psi} = \frac{2X(U-X)\left(-1 + \dfrac{Y_T^2 - 2(U-X)^2}{RAD}\right)}{D^2} \tag{50}$$

$$Y_T Y_L \frac{\partial \psi}{\partial r} = \frac{Y_L^2}{Y} \frac{\partial Y}{\partial r} - \left(V_r \frac{\partial Y_r}{\partial r} + V_\theta \frac{\partial Y_\theta}{\partial r} + V_\varphi \frac{\partial Y_\varphi}{\partial r} \right) \left(\frac{Y_L}{V} \right) \tag{51}$$

$$Y_T Y_L \frac{\partial \psi}{\partial \theta} = \frac{Y_L^2}{Y} \frac{\partial Y}{\partial \theta} - \left(V_r \frac{\partial Y_r}{\partial \theta} + V_\theta \frac{\partial Y_\theta}{\partial \theta} + V_\varphi \frac{\partial Y_\varphi}{\partial \theta} \right) \left(\frac{Y_L}{V} \right) \tag{52}$$

$$Y_T Y_L \frac{\partial \psi}{\partial \varphi} = \frac{Y_L^2}{Y} \frac{\partial Y}{\partial \varphi} - \left(V_r \frac{\partial Y_r}{\partial \varphi} + V_\theta \frac{\partial Y_\theta}{\partial \varphi} + V_\varphi \frac{\partial Y_\varphi}{\partial \varphi} \right) \left(\frac{Y_L}{V} \right) \tag{53}$$

$$n \frac{\partial n}{\partial X} = - \frac{\left(2U(U-X)^2 - Y_T^{\,2}(U-2X) + \dfrac{Y_T^4 (U-2X) + 4Y_L^2 (U-X)^3}{RAD} \right)}{D^2} \tag{54}$$

$$n \frac{\partial n}{\partial Y} = \frac{2X(U-X)}{D^2\ Y} \left(-Y_T^2 + \frac{Y_T^4 + 2Y_L^2 (U-X)^2}{RAD} \right) \tag{55}$$

$$n \frac{\partial n}{\partial Z} = \frac{iX}{D^2} \left(-2(U-X)^2 - Y_T^2 + \frac{Y_T^4}{RAD} \right) \tag{56}$$

$$n \frac{\partial n}{\partial r} = n \frac{\partial n}{\partial X} \frac{\partial X}{\partial r} + n \frac{\partial n}{\partial Y} \frac{\partial Y}{\partial r} + n \frac{\partial n}{\partial Z} \frac{\partial Z}{\partial r} + \frac{n}{Y_L Y_T} \frac{\partial n}{\partial \psi} Y_L Y_T \frac{\partial \psi}{\partial r} \tag{57}$$

$$n \frac{\partial n}{\partial \theta} = n \frac{\partial n}{\partial X} \frac{\partial X}{\partial \theta} + n \frac{\partial n}{\partial Y} \frac{\partial Y}{\partial \theta} + n \frac{\partial n}{\partial Z} \frac{\partial Z}{\partial \theta} + \frac{n}{Y_L Y_T} \frac{\partial n}{\partial \psi} Y_T Y_L \frac{\partial \psi}{\partial \theta} \tag{58}$$

$$n \frac{\partial n}{\partial \varphi} = n \frac{\partial n}{\partial X} \frac{\partial X}{\partial \varphi} + n \frac{\partial n}{\partial Y} \frac{\partial Y}{\partial \varphi} + n \frac{\partial n}{\partial Z} \frac{\partial Z}{\partial \varphi} + \frac{n}{Y_L Y_T} \frac{\partial n}{\partial \psi} Y_T Y_L \frac{\partial \psi}{\partial \varphi} \tag{59}$$

$$n \frac{\partial n}{\partial V_r} = \frac{n}{Y_T Y_L} \frac{\partial n}{\partial \psi} \left(\frac{V_r Y_L^2}{V^2} - \left(\frac{Y_L}{V} \right) Y_r \right) \tag{60}$$

$$n\frac{\partial n}{\partial V_\theta} = \frac{n}{Y_L Y_T}\frac{\partial n}{\partial \psi}\left(\frac{V_\theta Y_L^2}{V^2} - \left(\frac{Y_L}{V}\right) Y_\theta\right) \tag{61}$$

$$n\frac{\partial n}{\partial V_\varphi} = \frac{n}{Y_L Y_T}\frac{\partial n}{\partial \psi}\left(\frac{V_\varphi Y_L^2}{V^2} - \left(\frac{Y_L}{V}\right) Y_\varphi\right) \tag{62}$$

$$nn' = n^2 - \left(2Xn\frac{\partial n}{\partial X} + Y\,n\frac{\partial n}{\partial Y} + Z\,n\frac{\partial n}{\partial Z}\right) \tag{63}$$

$$\text{Polarization} = \rho = -i\,\frac{\left(-Y_T^2 + \text{RAD}\right)\sqrt{V^2}}{2(U-X)\,V\cdot Y} \tag{64a}$$

(Budden, 1961, page 49).

$$\text{Longitudinal polarization} = \frac{iX\sqrt{Y_T^2}}{(U-X)\left(U+i\,\frac{V\cdot Y}{\sqrt{V^2}}\,\rho\right)} \tag{64b}$$

(Budden, 1961, page 54).

$$n\frac{\partial n}{\partial t} = n\frac{\partial n}{\partial X}\frac{\partial X}{\partial t} \tag{65}$$

5.2 Appleton-Hartree Formula with Field, no Collisions

The equations are the same as with field with collisions except for:

$$Z = \frac{\partial Z}{\partial r} = \frac{\partial Z}{\partial \theta} = \frac{\partial Z}{\partial \varphi} = 0, \tag{66}$$

$$U = 1. \tag{67}$$

5.3 Appleton-Hartree Formula no Field, With Collisions

$$U = 1 - iZ \tag{68}$$

$$n^2 = 1 - \frac{X}{U} \tag{69}$$

$$n\frac{\partial n}{\partial X} = -\frac{.5}{U} \tag{70}$$

$$n\frac{\partial n}{\partial Z} = -\frac{.5iX}{U^2} \tag{71}$$

$$nn' = n^2 - \left(2X\,n\frac{\partial n}{\partial X} + Z\,n\frac{\partial n}{\partial Z}\right) \tag{72}$$

$$n\frac{\partial n}{\partial r} = n\frac{\partial n}{\partial X}\frac{\partial X}{\partial r} + n\frac{\partial n}{\partial Z}\frac{\partial Z}{\partial r} \tag{73}$$

$$n\frac{\partial n}{\partial \theta} = n\frac{\partial n}{\partial X}\frac{\partial X}{\partial \theta} + n\frac{\partial n}{\partial Z}\frac{\partial Z}{\partial \theta} \tag{74}$$

$$n\frac{\partial n}{\partial \varphi} = n\frac{\partial n}{\partial X}\frac{\partial X}{\partial \varphi} + n\frac{\partial n}{\partial Z}\frac{\partial Z}{\partial \varphi} \tag{75}$$

$$n\frac{\partial n}{\partial V_r} = 0 \tag{76}$$

$$n\frac{\partial n}{\partial V_\theta} = 0 \tag{77}$$

$$n\frac{\partial n}{\partial V_\varphi} = 0 \tag{78}$$

Polarization = i (79a)

Longitudinal polarization = 0 (79b)

$$n\frac{\partial n}{\partial t} = n\frac{\partial n}{\partial X}\frac{\partial X}{\partial t} \tag{80}$$

5.4 Appleton-Hartree Formula no Field, no Collisions

$$n^2 = 1 - X \tag{81}$$

$$n\frac{\partial n}{\partial X} = -.5 \tag{82}$$

$$n\frac{\partial n}{\partial r} = \left(n\frac{\partial n}{\partial X}\right)\left(\frac{\partial X}{\partial r}\right) \tag{83}$$

$$n\frac{\partial n}{\partial \theta} = \left(n\frac{\partial n}{\partial X}\right)\frac{\partial X}{\partial \theta} \tag{84}$$

$$n\frac{\partial n}{\partial \varphi} = \left(n\frac{\partial n}{\partial X}\right)\frac{\partial X}{\partial \varphi} \tag{85}$$

$$n\frac{\partial n}{\partial V_r} = 0 \tag{86}$$

$$n\frac{\partial n}{\partial V_\theta} = 0 \tag{87}$$

$$n\frac{\partial n}{\partial V_\varphi} = 0 \tag{88}$$

$$nn' = 1 \tag{89}$$

$$\text{Polarization} = i \tag{90a}$$

$$\text{Longitudinal polarization} = 0 \tag{90b}$$

$$n\frac{\partial n}{\partial t} = n\frac{\partial n}{\partial X}\frac{\partial X}{\partial t} \tag{91}$$

5.5 Booker Quartic with Field, with Collisions

The form of the dispersion relation used for the Hamiltonian in this version of subroutine RINDEX is the quadratic equation whose solution is the Appleton-Hartree formula. This Hamiltonian, given by

(22), is also a special case of the Booker quartic for S = 0 and C = 1 (Budden, 1961). This version uses the Hamiltonian in (22) only when the electron density is large enough that X is greater than or equal to 1/10. For X less than 1/10, the Hamiltonian in (21) is used.

Below are the equations for the derivatives of the Hamiltonian in (22). The equations for the derivatives of the Hamiltonian in (21) are the same as those in section 4.1.

$$k^2 = k_r^2 + k_\theta^2 + k_\varphi^2 \quad , \tag{92}$$

$$k \cdot Y = k_r Y_r + k_\theta Y_\theta + k_\varphi Y_\varphi \quad , \tag{93}$$

$$U = 1 - iZ \quad , \tag{94}$$

$$A = (U - X)\, U^2 - UY^2 \quad , \tag{95}$$

$$B = -2U(U - X)^2 + Y^2(2U - X) \quad , \tag{96}$$

$$\alpha = A\, c^4 k^4 + X(k \cdot Y)^2\, c^4 k^2 \quad , \tag{97}$$

$$\beta = B\, c^2 k^2 \omega^2 - X(k \cdot Y)^2\, c^2 \omega^2 \quad , \tag{98}$$

$$\gamma = ((U - X)^2 - Y^2)(U - X)\, \omega^4 \quad , \tag{99}$$

$$H = \alpha + \beta + \gamma \quad , \tag{100}$$

$$\frac{\partial H}{\partial X} = -U^2 c^4 k^4 + (k \cdot Y)^2 c^4 k^2 + (4U(U - X) - Y^2) c^2 k^2 \omega^2 +$$

$$-(k \cdot Y)^2 c^2 \omega^2 + (-3(U - X)^2 + Y^2)\, \omega^4 \quad , \tag{101}$$

$$\frac{\partial H}{\partial (Y^2)} = -U\, c^4 k^4 + (2U - X)\, c^2 k^2 \omega^2 - (U - X)\, \omega^4 \quad , \tag{102}$$

$$\frac{\partial H}{\partial ((k \cdot Y)^2)} = X\, c^2 (c^2 k^2 - \omega^2) \quad , \tag{103}$$

$$\begin{aligned} \frac{\partial H}{\partial U} = {} & (2\, U(U - X) + U^2 - Y^2)\, c^4 k^4 + \\ & + 2(Y^2 - (U - X)^2 - 2U(U - X))\, c^2 k^2 \omega^2 + \\ & + (3(U - X)^2 - Y^2)\, \omega^4 \quad , \end{aligned} \tag{104}$$

$$\frac{\partial H}{\partial Z} = -i\, \frac{\partial H}{\partial U} \quad , \tag{105}$$

$$\frac{\partial H}{\partial (k^2)} = 2\, A\, c^4 k^2 + X (k \cdot Y)^2\, c^4 + B c^2 \omega^2 \quad , \tag{106}$$

$$\frac{\partial H}{\partial t} = \frac{\partial H}{\partial X} \frac{\partial X}{\partial t} \quad , \tag{107}$$

$$\begin{aligned} \frac{\partial H}{\partial r} = {} & \frac{\partial H}{\partial X} \frac{\partial X}{\partial r} + 2\, \frac{\partial H}{\partial (Y^2)}\, Y \frac{\partial Y}{\partial r} + \frac{\partial H}{\partial Z} \frac{\partial Z}{\partial r} + \\ & + 2\, \frac{\partial H}{\partial ((k \cdot Y)^2)} (k \cdot Y) \left(k_r \frac{\partial Y_r}{\partial r} + k_\theta \frac{\partial Y_\theta}{\partial r} + k_\varphi \frac{\partial Y_\varphi}{\partial r} \right) \quad , \end{aligned} \tag{108}$$

$$\begin{aligned} \frac{\partial H}{\partial \theta} = {} & \frac{\partial H}{\partial X} \frac{\partial X}{\partial \theta} + 2\, \frac{\partial H}{\partial (Y^2)}\, Y \frac{\partial Y}{\partial \theta} + \frac{\partial H}{\partial Z} \frac{\partial Z}{\partial \theta} + \\ & + 2\, \frac{\partial H}{\partial ((k \cdot Y)^2)} (k \cdot Y) \left(k_r \frac{\partial Y_r}{\partial \theta} + k_\theta \frac{\partial Y_\theta}{\partial \theta} + k_\varphi \frac{\partial Y_\varphi}{\partial \theta} \right) \quad , \end{aligned} \tag{109}$$

$$\frac{\partial H}{\partial \varphi} = \frac{\partial H}{\partial X}\frac{\partial X}{\partial \varphi} + 2\frac{\partial H}{\partial (Y^2)} Y \frac{\partial Y}{\partial \varphi} + \frac{\partial H}{\partial Z}\frac{\partial Z}{\partial \varphi} +$$

$$+ 2\frac{\partial H}{\partial ((k\cdot Y)^2)}(k\cdot Y)\left(k_r \frac{\partial Y_r}{\partial \varphi} + k_\theta \frac{\partial Y_\theta}{\partial \varphi} + k_\varphi \frac{\partial Y_\varphi}{\partial \varphi}\right) , \quad (110)$$

$$\frac{\partial H}{\partial \omega} = (2\beta + 4\gamma)/\omega - 2\frac{\partial H}{\partial X}\frac{X}{\omega} - 2\frac{\partial H}{\partial (Y^2)}\frac{Y^2}{\omega} +$$

$$- 2\frac{\partial H}{\partial ((k\cdot Y)^2)}\frac{(k\cdot Y)^2}{\omega} - \frac{\partial H}{\partial Z}\frac{Z}{\omega} , \quad (111)$$

$$\frac{\partial H}{\partial k_r} = 2\frac{\partial H}{\partial (k^2)} k_r + 2(k\cdot Y)\frac{\partial H}{\partial ((k\cdot Y)^2)} Y_r , \quad (112)$$

$$\frac{\partial H}{\partial k_\theta} = 2\frac{\partial H}{\partial (k^2)} k_\theta + 2(k\cdot Y)\frac{\partial H}{\partial ((k\cdot Y)^2)} Y_\theta , \quad (113)$$

$$\frac{\partial H}{\partial k_\varphi} = 2\frac{\partial H}{\partial (k^2)} k_\varphi + 2(k\cdot Y)\frac{\partial H}{\partial ((k\cdot Y)^2)} Y_\varphi , \quad (114)$$

$$k^2(\text{calculated}) = k^2 \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha} , \quad (115)$$

$$k\cdot\frac{\partial H}{\partial k} = k_r\frac{\partial H}{\partial k_r} + k_\theta\frac{\partial H}{\partial k_\theta} + k_\varphi\frac{\partial H}{\partial k_\varphi} = 4\alpha + 2\beta , \quad (116)$$

$$\text{Polarization} = i\frac{\sqrt{k^2}}{k\cdot Y}\left(U + \frac{X\omega^2}{c^2 k^2_{\text{calculated}} - \omega^2}\right) , \quad (117)$$

$$\text{Longitudinal Polarization} = i\frac{\sqrt{Y^2 - \frac{(k\cdot Y)^2}{k^2}}}{U - X}\left(1 - \frac{c^2 k^2_{\text{calculated}}}{\omega^2}\right) . \quad (118)$$

5.6 Booker Quartic with Field, no Collisions

All the equations here are the same as for the Booker quartic version with collisions (section 5.5) except for:

$$Z = \frac{\partial Z}{\partial r} = \frac{\partial Z}{\partial \theta} = \frac{\partial Z}{\partial \varphi} = 0 \ , \tag{119}$$

$$U = 1. \tag{120}$$

All the variables except polarization are real; polarization is pure imaginary.

5.7 Sen-Wyller Formula with Field

It is possible to write the generalized Appleton-Hartree formula of Sen and Wyller (1960) in the following form:

$$n^2 = 1 - \frac{2X(U-X) + 2\,AUX\,\sin^2\psi}{2U(U-X)(1+A) + 2\,AUX\,\sin^2\psi - U((1-BC)U + A(U+X))\,\sin^2\psi + RAD} \ , \tag{121}$$

where

$$RAD = \pm\sqrt{U^2((1-BC)U + A(U+X))^2\,\sin^4\psi + U^2(U-X)^2(C-B)^2\cos^2\psi} \ , \tag{122}$$

$$A = \frac{C+B}{2} - 1 \ , \tag{123}$$

$$B = \frac{F\left(\frac{1}{Z}\right)}{F\left(\frac{1-Y}{Z}\right)} \ , \tag{124}$$

$$C = \frac{F\left(\frac{1}{Z}\right)}{F\left(\frac{1+Y}{Z}\right)} , \tag{125}$$

$$U = \frac{Z}{F\left(\frac{1}{Z}\right)} \tag{126}$$

$$F(w) = w\, C_{3/2}(w) + i\,\frac{5}{2}\, C_{5/2}(w) = \frac{1}{\frac{3}{2}!} \int_0^\infty \frac{t^{3/2} e^{-t} dt}{w - it} , \tag{127}$$

$$X = \frac{f_N^2}{f^2} , \tag{128}$$

$$Y = \frac{f_H}{f} , \tag{129}$$

$$Z = \frac{\nu_m}{2\pi f} , \tag{130}$$

f_N is the plasma frequency, f_H is the electron gyrofrequency, ν_m is the mean electron collision frequency, f is the wave frequency, and ψ is the angle between the wave normal direction and the earth's magnetic field. (Note that if we would use

$$F(w) = \frac{1}{w - i} , \tag{131}$$

then (121) would reduce to the usual Appleton-Hartree formula.)

This version of RINDEX calls subroutine FSW. The first argument in the calling sequence is the argument w of F(w). The second argument in the calling sequence is the value of the function F(w) calculated by FSW. The third argument in the calling sequence is the derivative of the function, F'(w).

The following equations parallel the formulas in this version of RINDEX.

$$\alpha = F\left(\frac{1}{Z}\right) \tag{132}$$

$$\alpha' = F'\left(\frac{1}{Z}\right) \tag{133}$$

$$\beta = F\left(\frac{1-Y}{Z}\right) \tag{134}$$

$$\beta' = F'\left(\frac{1-Y}{Z}\right) \tag{135}$$

$$\gamma = F\left(\frac{1+Y}{Z}\right) \tag{136}$$

$$\gamma' = F'\left(\frac{1+Y}{Z}\right) \tag{137}$$

$$U = \frac{Z}{\alpha} \tag{138}$$

$$\frac{\partial U}{\partial Z} = \left(1 + \frac{\alpha'}{\alpha Z}\right) \Big/ \alpha \tag{139}$$

$$B = \frac{\alpha}{\beta} \tag{140}$$

$$\frac{\partial B}{\partial Y} = \frac{B}{Z}\ \frac{\beta'}{\beta} \tag{141}$$

$$\frac{\partial B}{\partial Z} = -\frac{B}{Z^2}\left(\frac{\alpha'}{\alpha} - (1-Y)\ \frac{\beta'}{\beta}\right) \tag{142}$$

$$C = \frac{\alpha}{\gamma} \tag{143}$$

$$\frac{\partial C}{\partial Y} = -\frac{C}{Z}\,\frac{\gamma'}{\gamma} \tag{144}$$

$$\frac{\partial C}{\partial Z} = -\frac{C}{Z^2}\left(\frac{\alpha'}{\alpha} - (1+Y)\,\frac{\gamma'}{\gamma}\right) \tag{145}$$

$$A = .5(B+C) - 1 \tag{146}$$

$$\frac{\partial A}{\partial Y} = .5\left(\frac{\partial B}{\partial Y} + \frac{\partial C}{\partial Y}\right) \tag{147}$$

$$\frac{\partial A}{\partial Z} = .5\left(\frac{\partial B}{\partial Z} + \frac{\partial C}{\partial Z}\right) \tag{148}$$

$$V^2 = V_r^2 + V_\theta^2 + V_\varphi^2 \tag{149}$$

$$V \cdot Y = V_r Y_r + V_\theta Y_\theta + V_\varphi Y_\varphi \tag{150}$$

$$Y_L^2 = \frac{(V \cdot Y)^2}{V^2} \tag{151}$$

$$Y_T^2 = Y^2 - Y_L^2 \tag{152}$$

$$\sin^2\psi = \frac{Y_T^2}{Y^2} \tag{153}$$

$$\cos^2\psi = \frac{Y_L^2}{Y^2} \tag{154}$$

$$T_1 = \left[(1-BC)\,U^2 + A\,U(U+X)\right] \sin^2\psi \tag{155}$$

$$\frac{\partial T_1}{\partial X} = +AU \sin^2\psi \tag{156}$$

$$\frac{\partial T_1}{\partial Y} = \left(U(U+X)\frac{\partial A}{\partial Y} - U^2\left(B\frac{\partial C}{\partial Y} + C\frac{\partial B}{\partial Y}\right)\right)\sin^2\psi \tag{157}$$

$$\frac{\partial T_1}{\partial Z} = \left(2U\frac{\partial U}{\partial Z}(1-BC+A) + A\,X\frac{\partial U}{\partial Z} - U^2\left(B\frac{\partial C}{\partial Z} + C\frac{\partial B}{\partial Z}\right) + U(U+X)\frac{\partial A}{\partial Z}\right)\sin^2\psi \tag{158}$$

$$\frac{1}{Y_L Y_T}\frac{\partial T_1}{\partial \psi} = \frac{2T_1}{Y_T^2} \tag{159}$$

$$T_2 = U^2\,(C-B)^2\,(U-X)^2\,\cos^2\psi \tag{160}$$

$$\frac{\partial T_2}{\partial X} = -2(U-X)\,U^2\,(C-B)^2\cos^2\psi \tag{161}$$

$$\frac{\partial T_2}{\partial Y} = 2U^2(U-X)^2\,\cos^2\psi\,(C-B)\left(\frac{\partial C}{\partial Y} - \frac{\partial B}{\partial Y}\right) \tag{162}$$

$$\frac{\partial T_2}{\partial Z} = 2U^2\,(U-X)^2(C-B)\left(\frac{\partial C}{\partial Z} - \frac{\partial B}{\partial Z}\right)\cos^2\psi + 2T_2\left(\frac{1}{U} + \frac{1}{U-X}\frac{\partial U}{\partial Z}\right) \tag{163}$$

$$\frac{1}{Y_L Y_T}\frac{\partial T_2}{\partial \psi} = -\frac{2T_2}{Y_L^2} \tag{164}$$

$$RAD = \pm\sqrt{T_1^2 + T_2} \tag{165}$$

$$\frac{\partial RAD}{\partial X} = \frac{T_1 \frac{\partial T_1}{\partial X} + \frac{1}{2} \frac{\partial T_2}{\partial X}}{RAD} \tag{166}$$

$$\frac{\partial RAD}{\partial Y} = \frac{T_1 \frac{\partial T_1}{\partial Y} + \frac{1}{2} \frac{\partial T_2}{\partial Y}}{RAD} \tag{167}$$

$$\frac{\partial RAD}{\partial Z} = \frac{T_1 \frac{\partial T_1}{\partial Z} + \frac{1}{2} \frac{\partial T_2}{\partial Z}}{RAD} \tag{168}$$

$$\frac{1}{Y_L Y_T} \frac{\partial RAD}{\partial \psi} = \frac{T_1 \left(\frac{1}{Y_L Y_T} \frac{\partial T_1}{\partial \psi} \right) + \frac{1}{2} \left(\frac{1}{Y_L Y_T} \frac{\partial T_2}{\partial \psi} \right)}{RAD} \tag{169}$$

$$D = 2U(U-X)(1 + A) - T_1 + RAD + 2AUX \sin^2 \psi \tag{170}$$

$$\frac{\partial D}{\partial X} = -2U - \frac{\partial T_1}{\partial X} + \frac{\partial RAD}{\partial X} + 2\, AU \sin^2 \psi \tag{171}$$

$$\frac{\partial D}{\partial Y} = 2U(U-X)\frac{\partial A}{\partial Y} - \frac{\partial T_1}{\partial Y} + \frac{\partial RAD}{\partial Y} + 2\, U \sin^2 \psi \frac{\partial A}{\partial Y} \tag{172}$$

$$\begin{aligned} \frac{\partial D}{\partial Z} = {} & 2(1+A) \frac{\partial U}{\partial Z} (2\, U-X) + 2U(U-X) \frac{\partial A}{\partial Z} - \frac{\partial T_1}{\partial Z} + \frac{\partial RAD}{\partial Z} + \\ & + 2AX \sin^2 \psi \frac{\partial U}{\partial Z} + 2\, UX \sin^2 \psi \frac{\partial A}{\partial Z} \end{aligned} \tag{173}$$

$$\frac{1}{Y_L Y_T} \frac{\partial D}{\partial \psi} = -\left(\frac{1}{Y_L Y_T} \frac{\partial T_1}{\partial \psi} \right) + \left(\frac{1}{Y_L Y_T} \frac{\partial RAD}{\partial \psi} \right) + 2\, AUX / Y^2 \tag{174}$$

$$n^2 - 1 = \frac{-2X}{D}\left((U-X) + U A \sin^2\psi \right) \tag{175}$$

$$n^2 = 1 + (n^2 - 1) \tag{176}$$

$$n\frac{\partial n}{\partial X} = \frac{1}{2}(n^2 - 1)\left(\frac{1}{X} - \frac{1}{D}\frac{\partial D}{\partial X}\right) + \frac{X}{D} \tag{177}$$

$$n\frac{\partial n}{\partial Y} = -\frac{X U \sin^2\psi}{D}\frac{\partial A}{\partial Y} - \frac{(n^2-1)}{2D}\frac{\partial D}{\partial Y} \tag{178}$$

$$n\frac{\partial n}{\partial Z} = -\frac{X}{D}(1+A\sin^2\psi)\frac{\partial U}{\partial Z} - \frac{XU}{D}\sin^2\psi\frac{\partial A}{\partial Z} - \frac{(n^2-1)}{2D}\frac{\partial D}{\partial Z} \tag{179}$$

$$\frac{n}{Y_T Y_L}\frac{\partial n}{\partial \psi} = -\frac{XUA}{Y^2 D} - \frac{(n^2-1)}{2D}\left(\frac{1}{Y_L Y_T}\frac{\partial D}{\partial \psi}\right) \tag{180}$$

$$Y_L Y_T \frac{\partial \psi}{\partial r} = \frac{Y_L^2}{Y}\frac{\partial Y}{\partial r} - \left(V_r\frac{\partial Y_r}{\partial r} + V_\theta\frac{\partial Y_\theta}{\partial r} + V_\varphi\frac{\partial Y_\varphi}{\partial r}\right)\frac{V\cdot Y}{V^2} \tag{181}$$

$$Y_L Y_T \frac{\partial \psi}{\partial \theta} = \frac{Y_L^2}{Y}\frac{\partial Y}{\partial \theta} - \left(V_r\frac{\partial Y_r}{\partial \theta} + V_\theta\frac{\partial Y_\theta}{\partial \theta} + V_\varphi\frac{\partial Y_\varphi}{\partial \theta}\right)\frac{V\cdot Y}{V^2} \tag{182}$$

$$Y_L Y_T \frac{\partial \psi}{\partial \varphi} = \frac{Y_L^2}{Y}\frac{\partial Y}{\partial \varphi} - \left(V_r\frac{\partial Y_r}{\partial \varphi} + V_\theta\frac{\partial Y_\theta}{\partial \varphi} + V_\varphi\frac{\partial Y_\varphi}{\partial \varphi}\right)\frac{V\cdot Y}{V^2} \tag{183}$$

$$n\frac{\partial n}{\partial r} = n\frac{\partial n}{\partial X}\frac{\partial X}{\partial r} + n\frac{\partial n}{\partial Y}\frac{\partial Y}{\partial r} + n\frac{\partial n}{\partial Z}\frac{\partial Z}{\partial r} + \left(\frac{n}{Y_L Y_T}\frac{\partial n}{\partial \psi}\right)\left(Y_L Y_T\frac{\partial \psi}{\partial r}\right) \tag{184}$$

$$n\frac{\partial n}{\partial \theta} = n\frac{\partial n}{\partial X}\frac{\partial X}{\partial \theta} + n\frac{\partial n}{\partial Y}\frac{\partial Y}{\partial \theta} + n\frac{\partial n}{\partial Z}\frac{\partial Z}{\partial \theta} + \left(\frac{n}{Y_L Y_T}\frac{\partial n}{\partial \psi}\right)\left(Y_L Y_T\frac{\partial \psi}{\partial \theta}\right) \tag{185}$$

$$n\frac{\partial n}{\partial \varphi} = n\frac{\partial n}{\partial X}\frac{\partial X}{\partial \varphi} + n\frac{\partial n}{\partial Y}\frac{\partial Y}{\partial \varphi} + n\frac{\partial n}{\partial Z}\frac{\partial Z}{\partial \varphi} + \left(\frac{n}{Y_L Y_T}\frac{\partial n}{\partial \psi}\right)\left(Y_L Y_T\frac{\partial \psi}{\partial \varphi}\right) \tag{186}$$

$$n\frac{\partial n}{\partial V_r} = \left(\frac{n}{Y_L Y_T}\ \frac{\partial n}{\partial \psi}\right)\left(V_r Y_L^2 - (V\cdot Y)Y_r\right) \Big/ V^2 \tag{187}$$

$$n\frac{\partial n}{\partial V_\theta} = \left(\frac{n}{Y_L Y_T}\ \frac{\partial n}{\partial \psi}\right)\left(V_\theta Y_L^2 - (V\cdot Y)Y_\theta\right) \Big/ V^2 \tag{188}$$

$$n\frac{\partial n}{\partial V_\varphi} = \left(\frac{n}{Y_L Y_T}\ \frac{\partial n}{\partial \psi}\right)\left(V_\varphi Y_L^2 - (V\cdot Y)Y_\varphi\right) \Big/ V^2 \tag{189}$$

$$nn' = n^2 - \left(2X\left(n\frac{\partial n}{\partial X}\right) + Y\left(n\frac{\partial n}{\partial Y}\right) + Z\left(n\frac{\partial n}{\partial Z}\right)\right) \tag{190}$$

$$\text{polarization} = \rho = \frac{i\,(T_1 - RAD)\,Y\sqrt{V^2}}{U(U-X)(C-B)(V\cdot Y)} \tag{191a}$$

$$\text{longitudinal polarization} = \frac{X(.5i(C-B)\rho + A\cos\psi)\sqrt{\sin^2\psi}}{\rho((U-X)(1+.5i(C-B)\rho\cos\psi)+A(U-X\cos^2\psi))} \tag{191b}$$

where

$$\cos\psi = \frac{V\cdot Y}{Y\sqrt{V^2}} \tag{192}$$

$$n\frac{\partial n}{\partial t} = n\frac{\partial n}{\partial X}\ \frac{\partial X}{\partial t} \tag{193}$$

5.8 Sen-Wyller Formula no Field

This subroutine uses the function G which is related to the function F (defined in the previous section) as follows:

$$G(w) = wF(w)\ . \tag{194}$$

The following equations parallel the formulas used in this version of RINDEX.

$$F_1 = F\left(\frac{1}{Z}\right) \tag{195}$$

$$F_1' = F'\left(\frac{1}{Z}\right) \tag{196}$$

$$G_1 = G\left(\frac{1}{Z}\right) \tag{197}$$

$$G_1' = G'\left(\frac{1}{Z}\right) \tag{198}$$

$$n^2 = 1 - X\,G_1 \tag{199}$$

$$n\frac{\partial n}{\partial X} = -\frac{1}{2}\,G_1 \tag{200}$$

$$n\frac{\partial n}{\partial Z} = \frac{X\,G_1'}{2\,Z^2} \tag{201}$$

$$n\frac{\partial n}{\partial r} = \left(n\frac{\partial n}{\partial X}\right)\frac{\partial X}{\partial r} + \left(n\frac{\partial n}{\partial Z}\right)\frac{\partial Z}{\partial r} \tag{202}$$

$$n\frac{\partial n}{\partial \theta} = \left(n\frac{\partial n}{\partial X}\right)\frac{\partial X}{\partial \theta} + \left(n\frac{\partial n}{\partial Z}\right)\frac{\partial Z}{\partial \theta} \tag{203}$$

$$n\frac{\partial n}{\partial \varphi} = \left(n\frac{\partial n}{\partial X}\right)\frac{\partial X}{\partial \varphi} + \left(n\frac{\partial n}{\partial Z}\right)\frac{\partial Z}{\partial \varphi} \tag{204}$$

$$nn' = n^2 - \left(2\,X\left(n\frac{\partial n}{\partial X}\right) + Z\left(n\frac{\partial n}{\partial Z}\right)\right) \tag{205}$$

$$\text{polarization} = i \tag{206a}$$

$$\text{longitudinal polarization} = 0 \tag{206b}$$

$$n\frac{\partial n}{\partial t} = n\frac{\partial n}{\partial X}\,\frac{\partial X}{\partial t} \tag{207}$$

6. IONOSPHERIC MODELS

When using the program, one must specify ionospheric models which define electron density, collision frequency (if the effect of collisions is being considered), and the earth's magnetic field (if its effects are being taken into account) as a function of position in space. Each of these three characteristics of the ionosphere is defined by a separate subroutine.

Appendices 3, 4, 5, and 6 contain descriptions, input parameter forms and listings of ionospheric models that now exist. These ionospheric models are not likely to cover the needs of everyone who wants to use the program. Anticipating this when we wrote the program, we made it possible to add models easily. The user may make up his own ionospheric models by simply writing subroutines to define electron density, collision frequency, and the earth's magnetic field (and their gradients) as a function of position in space in spherical polar coordinates, following the form of the subroutines in appendices 3, 4, 5, and 6.

Appendix 3 contains electron density models; appendix 4 contains models of irregularities which may be applied as perturbations to any of the electron density models; appendix 5 contains models of the earth's magnetic field; and appendix 6 contains collision frequency models.

Having several versions of the subroutines for refractive index, electron density, collision frequency, and the earth's magnetic field gives the user not only a wide choice among ionospheric models, but also a variety of compromises between cost and an accurate description of the ionosphere, while still keeping the program simple.

7. FINDING THE RAY PATHS THAT CONNECT A TRANSMITTER AND RECEIVER

The reason for using a ray tracing program is to find all important ray paths that connect a given transmitter and receiver (either or both of which may be on a satellite) on a particular frequency and such properties of these ray paths as group time delay, phase time delay, and absorption of the wave. "All important" ray paths include those that reflect from the various ionospheric layers (including multiple reflections) and that propagate off the great circle path.

Since basically all that a ray tracing program can do is to calculate the path of a ray when given the transmitter location, frequency, and direction of transmission, it cannot directly calculate those ray paths that arrive at a specified receiver. The problem is to know, before tracing the ray, in which directions to transmit the ray so that it will arrive at the receiver. Since there are no general solutions to this problem, the user of a ray tracing program must rely on some sort of trial and error technique to find those ray paths that connect the transmitter and receiver. This involves varying the direction of transmission until a ray is found that reaches the receiver. If a ray tracing program does this automatically, we say that it has a homing feature. This program does not have such a feature. To find all the paths connecting the transmitter with the receiver requires a very elaborate homing routine because "homing in" on a receiver takes more judgment and common sense than speed in performing massive calculations. Therefore, the person using the program is more fitted to this task than is the computer program itself.

As an aid, however, the program allows the user to specify the receiver height, the number of hops, and a range of azimuth and elevation angles-of-transmission that he thinks will include those rays that

will arrive at the receiver. The program then calculates a ray path for each of the azimuth and elevation angles-of-transmission specified in the range. Usually only in the case of ionospheres with large horizontal gradients will the azimuth angle-of-transmission have to be varied. The program will calculate each ray path far enough to intersect or make a closest approach to the receiver height for the requested number of hops. The user can then interpolate between those rays which surround the receiver.

We define the point of "closest approach" as the point on the ray path where the wave normal direction is horizontal. It approximates an apogee if the receiver is above the apogee height and it approximates a perigee if the receiver is below the perigee height. The approximation is good for oblique propagation. When the earth's magnetic field is neglected, a point of "closest approach" is exactly an apogee or perigee.

We count one hop every time the ray crosses the receiver height. If the receiver is on the ground, a ground reflection counts as one hop for the downcoming ray before the ground reflection, and another hop for the upgoing ray after the ground reflection. We count two hops every time the ray passes through a point of "closest approach" to the receiver height. This procedure helps make rays that have the same hop number have a ground range that is a continuous function of the direction of transmission.

8. OUTPUT

8.1 Printout

Periodically and at selected points during a ray trace, the program will print information giving the position of the current ray path point, the direction of the wave normal, and the cumulative values of quantities being integrated along the ray path such as group path, phase path, absorption, and Doppler shift. Appendix 8c contains a sample of the printout.

8.2 Punched Cards

The program will punch a card at the beginning of each ray, a card at each ground reflection, a card at each crossing of the receiver height, two cards for each closest approach to the receiver height, and a card at the end of each ray to summarize the main results of the ray path calculations. These cards are explained in figures 1 and 2.

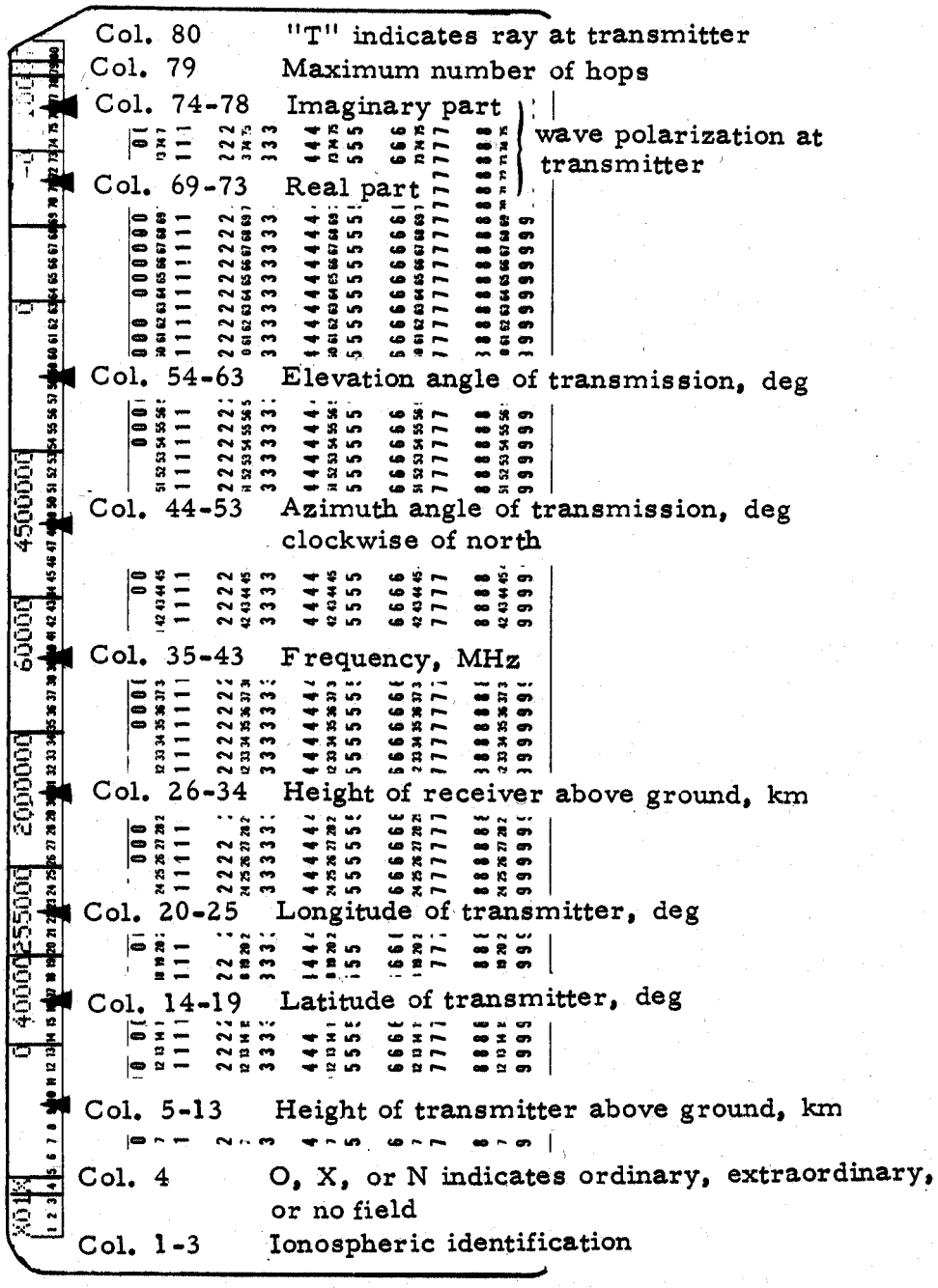
These cards are very useful as input data to other computer programs and for plotting the results of the ray tracing. In fact, these cards represent the most useful form of output for production ray path calculations. This method, called the rayset information-storage technique, was developed by Dr. T. A. Croft (Croft and Gregory, 1963) of Stanford University.

8.3 Plots of the Ray Path

A plot of the actual ray path, especially for very irregular ionospheres, can be helpful in understanding what sometimes seems like strange results in light of the input data. Thus, the program has an option for plotting, providing, of course, that the user has a plotter and plotting subroutines such as those described in appendix 7. The program can plot the projection of the ray path on any vertical plane or on the ground. The input parameter forms for plots of the ray path (appendix 1i) give more details. Appendix 8e contains sample plots of the raypath.

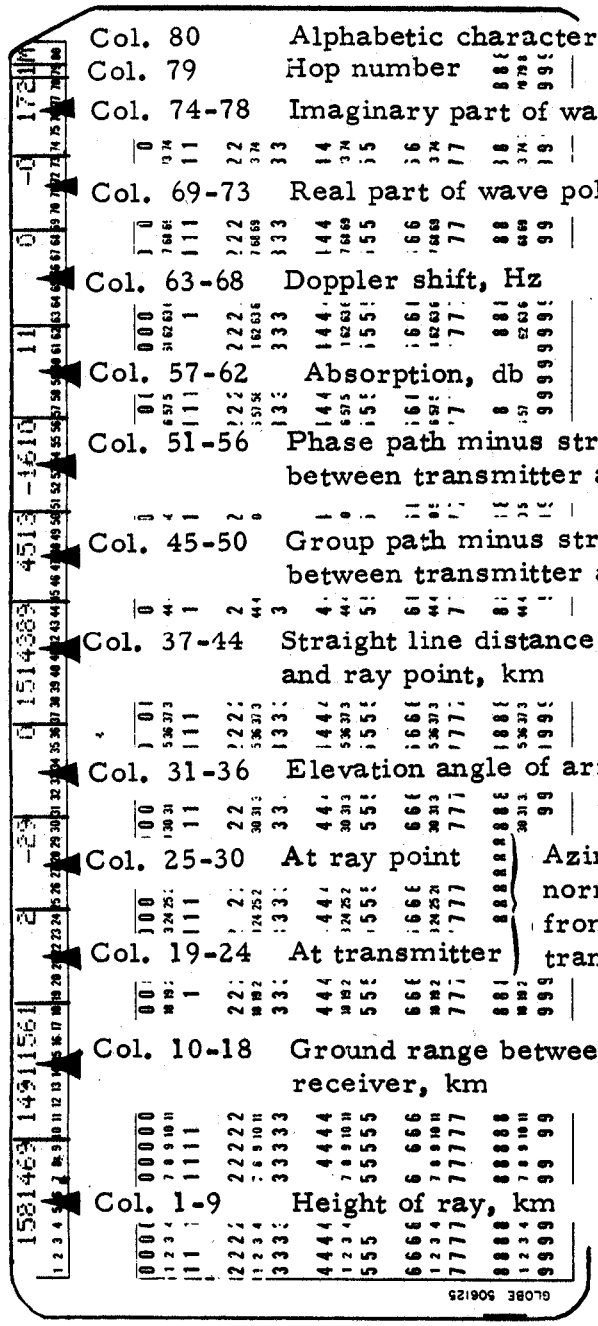
9. DECK SET UP

The versatility gained by having several versions of some of the subroutines is somewhat offset because the user must learn the deck set up in order to make necessary substitutions. Figure 3 shows the deck setup, including the subroutines that make up the main deck and those which are frequently exchanged with alternate versions. The order of the subroutines is unimportant.



▲ indicates implied decimal

Figure 1. Sample transmitter rayset.



*G ray ground reflected. The height punched out is the apogee height since the last ground reflect.
M ray made a closest approach to the receiver height
P ray penetrated
R ray at the receiver height. The height punched out is the height of the ray farthest from receiver height since last crossing of receiver height
S program reached maximum number of steps

Figure 2. Sample minimum distance rayset.

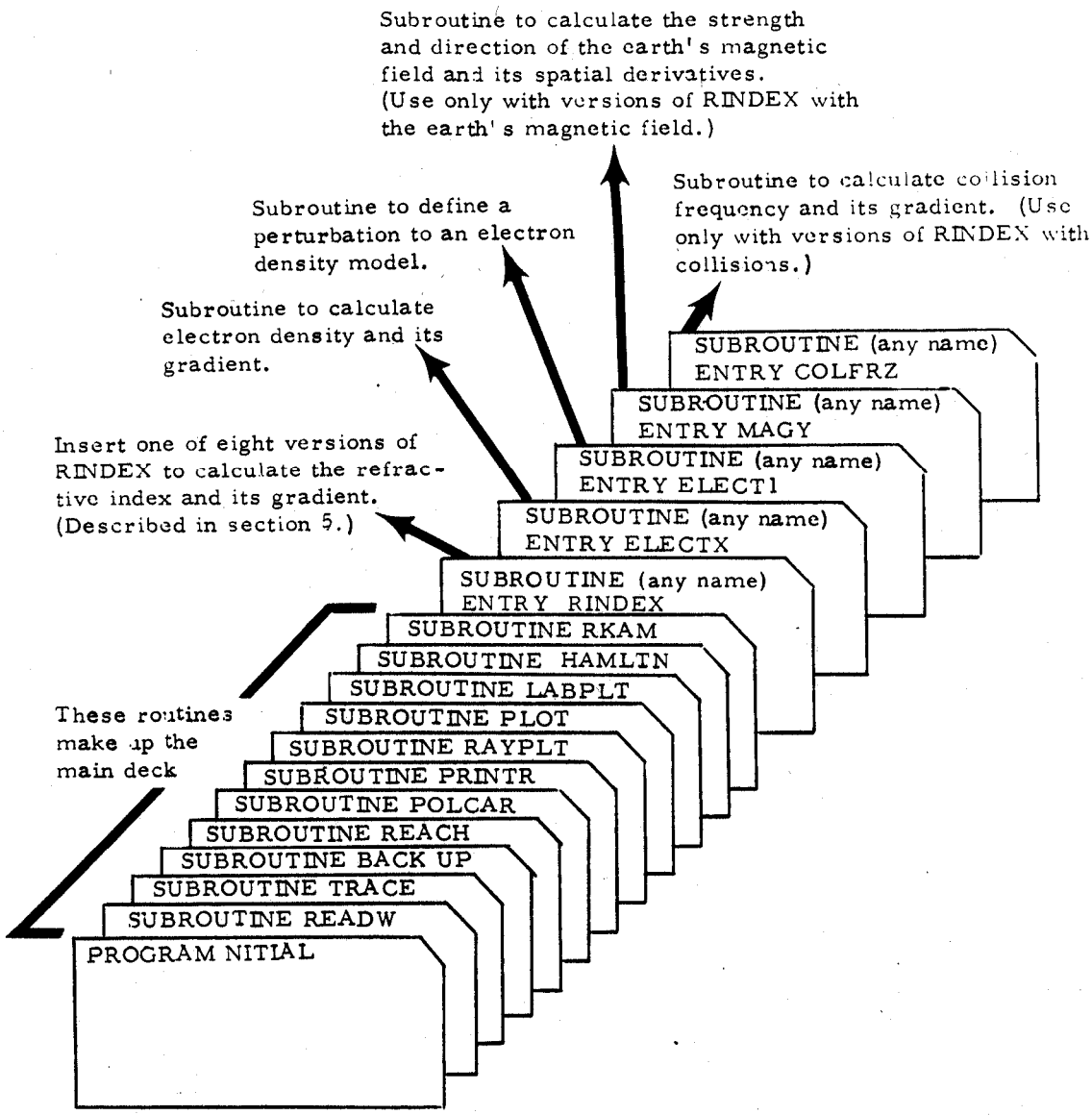


Figure 3. Program deck set-up

10. INPUT

The input data for a ray tracing program divide themselves naturally into two groups:

First, data that control the type of ray trace requested, such as the transmitter location and frequency, plus parameters describing analytic models of the ionosphere. Since there are few of these, efficiency in packing such data can be exchanged for versatility and ease of data handling. Therefore, by putting only one piece of data on each card, we gain the conveniences of reading in these data in any order and of having the program read in only those data that are different from those of the previous case. A number in the first three columns of each card identifies the data being read in. Table 2 defines the identifying numbers that are subscripts for a linear array, W. The last 56 columns of the card are available for comments.

We have also provided a method for conversion of units for input. The computer program needs angles in radians, whereas people usually like to use angles in degrees. The program is set up for angles in radians, but putting a "1" in column 18 allows the user to enter the angle in degrees and have the program make the conversion. A "1" in column 19 allows the user to enter central earth angles as the great circle distance along the ground in kilometers. (The program will calculate the latitude of a transmitter which is 500 km north of the equator, for instance.) The program expects distances in kilometers. A "1" in column 20 indicates a distance in nautical miles, and a "1" in column 21 indicates a distance in feet.

Appendix 8b contains a sample of how the cards are to be punched. If two or more cards have the same identifying number, the last one dominates. A card with the first three columns blank indicates the end of this type of data cards.

Table 2. Description of the Input Data for the W Array

W1	= 1. for ordinary ray
	= -1. for extraordinary ray
W2*	Radius of the earth in km
W3	Height of transmitter above the earth in km
W4	North geographic latitude of the transmitter
W5	East geographic longitude of the transmitter
W7	Initial frequency in MHz
W8	Final frequency in MHz
W9	Step in frequency in MHz (zero for a fixed frequency)
W11	Initial azimuth angle of transmission
W12	Final azimuth angle of transmission
W13	Step in azimuth angle of transmission (zero for a fixed azimuth)
W15	Initial elevation angle of transmission
W16	Final elevation angle of transmission
W17	Step in elevation angle of transmission (zero for a fixed elevation)
W20	Receiver height above the earth in km
W21	Nonzero to skip to the next frequency after the ray has penetrated the ionosphere
W22*	Maximum number of hops
W23*	Maximum number of steps per hop
W24*	North geographic latitude of the north geomagnetic pole
W25*	East geographic longitude of north geomagnetic pole
W41*	=1. for Runge-Kutta integration
	=2. for Adams-Moulton integration without error checking
	=3. for Adams-Moulton integration with relative error check
	=4. for Adams-Moulton integration with absolute error check
W42*	Maximum allowable single step error
W43*	Ratio of maximum single step error to minimum single step error
W44*	Initial integration step size in km (step in group path)
W45*	Maximum step length in km
W46*	Minimum step length in km
W47*	Factor by which to increase or decrease step length
W57	=1. to integrate, =2. to integrate and print phase path
W58	=1. to integrate, =2. to integrate and print absorption
W59	=1. to integrate, =2. to integrate and print doppler shift
W60	=1. to integrate, =2. to integrate and print path length
W71	Number of steps between periodic printout
W72	Nonzero to punch raysets on cards
W81	=0. to not plot ray path
	=1. to plot projection of ray path on a vertical plane
	=2. to plot projection of ray path on the ground
W82-88	Parameters used when plotting
W100-149	Parameters for analytic electron density models
W150-199	Parameters for perturbations to electron density models
W200-249	Parameters for analytic magnetic field models
W250-299	Parameters for analytic collision frequency models

*These values have been initialized in the main program but may be reset by reading them in. See Appendix 1b for the initial values.

A second group of input data cards are necessary if nonanalytic ionospheric models such as the electron density profile defined by subroutine TABLEX or the collision frequency profile defined by subroutine TABLEZ are used. Each subroutine defining a nonanalytic ionospheric model reads in data cards according to a format defined in that subroutine. An element in the W array controls the reading of these cards. (See table 2.) Figure 4 shows the order in which these data cards should be arranged.

11. ACCURACY

The numerical integration subroutine has a built-in mechanism to check errors and adjust the integration step length accordingly. If the errors get larger than a maximum specified by the user, the routine will decrease the step length in order to maintain the accuracy. On the other hand, if the accuracy is greater than that required by the user, the routine will increase the step length in order to reduce the computing cost. The user specifies the desired accuracy in W42 (see table 2). W42 is the maximum allowable relative error in any single step for any of the equations being integrated. To get a very accurate (but expensive) ray trace, one can use a small W42 (about 10^{-5} or 10^{-6}). For a cheap, approximate ray trace, one should use a large W42 (10^{-3} or even 10^{-2}). For cases in which all of the variables being integrated increase monotonically, the total relative error can be guaranteed to be less than W42. Otherwise, the total relative error cannot be easily estimated.

The far left column of the printout from the ray path calculation gives an indication of the integration error in the magnitude of the vector which points in the wave normal direction. Although the calculation of this error is made independently of the error calculation in the numerical integration routine, we have found that except near reflection for vertical or near vertical incidence this error is usually of the same order

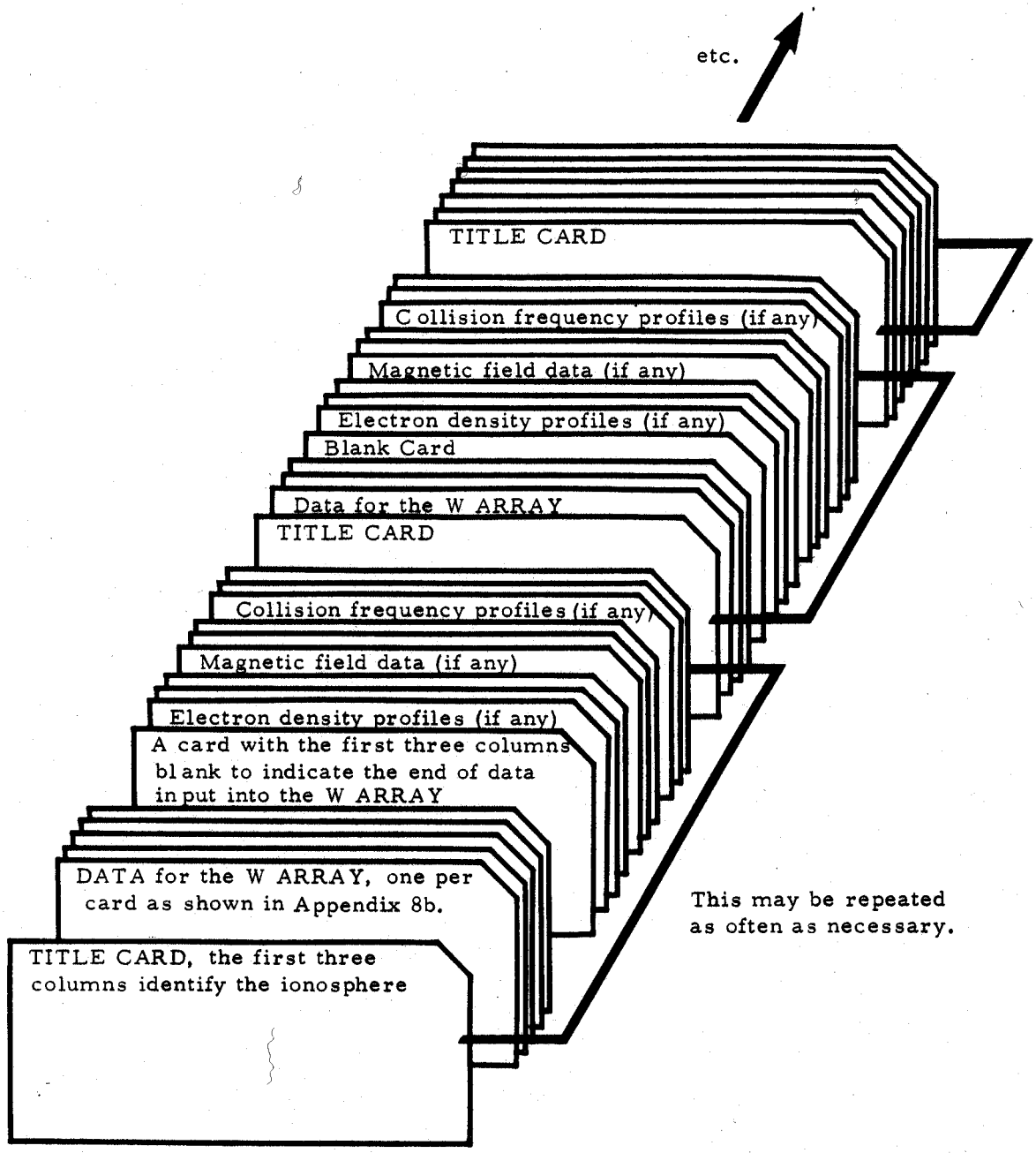


Figure 4. Data deck set-up.

of magnitude as that specified in W42. We have found that whenever this error has exceeded W42 by several orders of magnitude, the electron density subroutine we had written was calculating a gradient of electron density inconsistent with the spatial variation of electron density being calculated. See the general description of electron density models in Appendix 3a for more information.

12. COORDINATE SYSTEMS

The program uses two different spherical polar coordinate systems, namely, a geographic and a computational coordinate system. Input data for the coordinates of the transmitter (W4 and W5) and input data for the coordinates of the north pole of the computational coordinate system (W24 and W25) are entered in geographic coordinates. (Putting W25 equal to 0° and W24 equal to 90° would superimpose the two north poles and equate the two coordinate systems.)

When the two coordinate systems do not coincide, the three types of ionospheric models calculate electron density, the earth's magnetic field, and collision frequency in terms of the computational coordinate system. In particular, the dipole model of the earth's magnetic field uses the axis of the computational coordinate system as the axis for the dipole field. Thus, when using this dipole model, the computational coordinate system is a geomagnetic coordinate system, and both electron density and collision frequency must be defined in geomagnetic coordinates. Dudziak (1961) describes the transformations between these coordinate systems.

13. HOW THE PROGRAM WORKS

This ray tracing program consists of various subroutines that perform specific tasks in calculating ray paths. This division of labor facilitates modifying the program to solve specific problems. Often it may be necessary to change only one or two subroutines to convert the program to a different use.

The main program (NITIAL) sets up the initial conditions (transmitter location, wave frequency, and direction of transmission) for each ray trace. In setting up the initial conditions for each ray trace, the main program (NITIAL) steps frequency, azimuth angle of transmission, and elevation angle of transmission. The details of the workings of NITIAL can be found in the flow chart in figure 5. Then subroutine TRACE calculates one ray path for the requested number of crossings of the specified receiver height. Subroutine TRACE is the heart of the ray tracing program. It is the most complicated subroutine included, but also the most important to understand. The flow chart in figure 6 should help to explain TRACE.

Subroutine RKAM integrates the differential equations numerically using an Adams-Moulton predictor-corrector method with a Runge-Kutta starter. Subroutine HAMLTN evaluates the differential equations to be integrated. Subroutine RINDEX calculates the phase refractive index and its gradients, the group refractive index, and the polarization. (Eight versions of subroutine RINDEX are included.) Subroutines ELECTX, ELECT1, MAGY, and COLFRZ calculate the ionospheric electron density, perturbations to the electron density (irregularities), the earth's magnetic field, and the electron collision frequency, respectively. Several versions of these four subroutines are included and it is easy to add more. Subroutine REACH calculates a straight-line segment of a ray path in free space between the earth and the

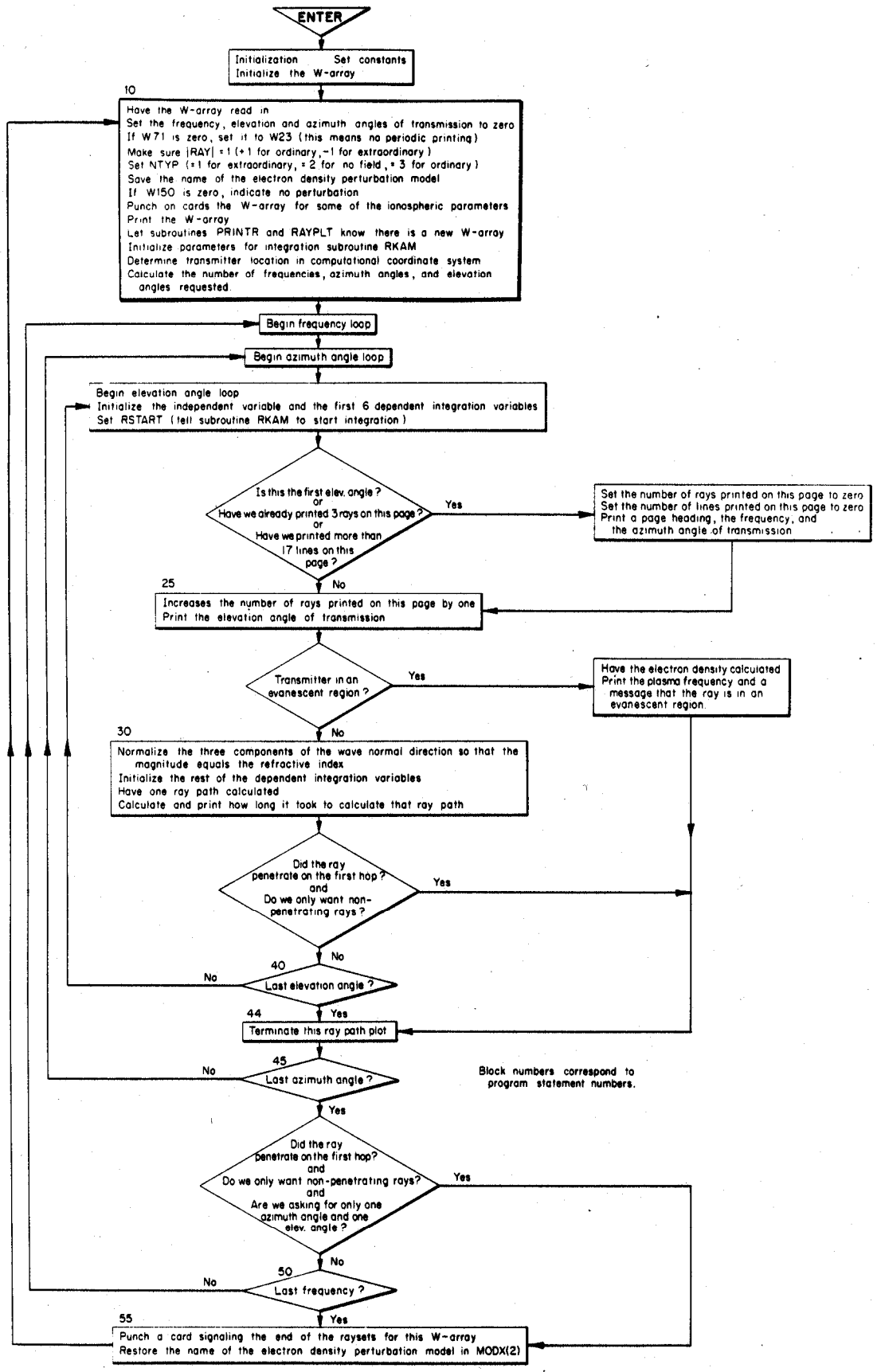


Figure 5. Flow chart for program NITIAL.

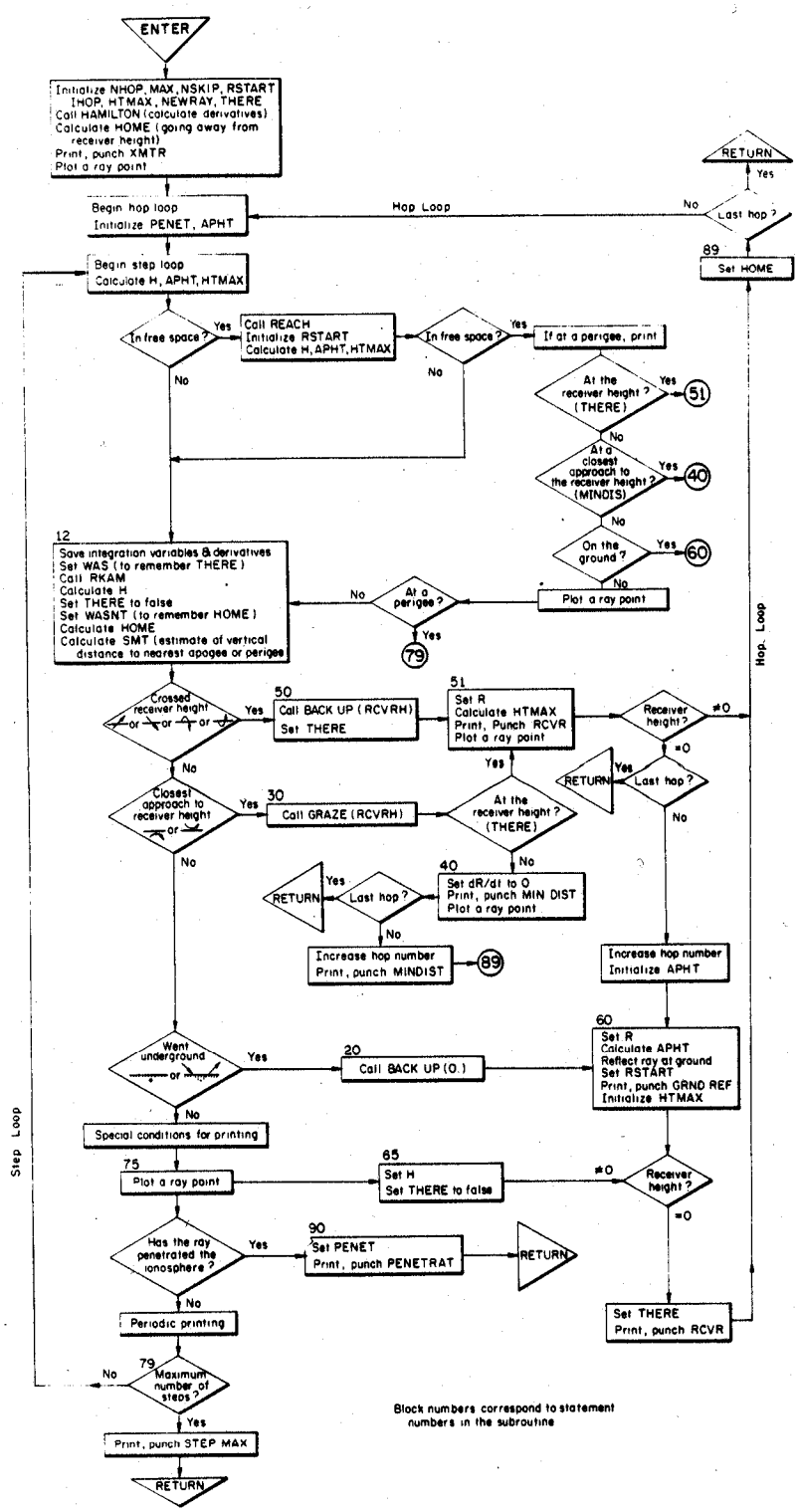


Figure 6. Flow chart for subroutine TRACE.

The ray graphics illustrate the path of a ray during a single step.

ionosphere or between ionospheric layers. Subroutine BACK UP finds an intersection of the ray with the receiver height or with the ground. Subroutine PRINTR prints information describing the ray path and punches the results on cards (raysets). Subroutine RAYPLT plots the ray path. The block diagram in figure 7 shows the relationship among these (and other) subroutines.

The listings of most of the subroutines have comments that should help in understanding how they work. In addition, Tables 3 through 14 define the variables in the common blocks.

14. ACKNOWLEDGMENTS

Part of the organization of this program into subroutines follows that of the program of Dudziak (1961), in particular for subroutines RKAM, HAMLTN , RINDEX, ELECTX, MAGY, and COLFRZ. Also, the coordinate transformation in subroutine PRINTR and the method for data input via the W array are taken from the program of Dudziak (1961). The term "rayset," the idea of punching results of each hop for each ray trace onto cards, and the idea of automatically plotting ray paths come from the program of Croft and Gregory (1963). The quasi-parabolic layer electron density model QPARAB is taken from the paper by Croft and Hoogasian (1968). Notice that the quasi-parabolic layer that is now in the program is slightly different from the one in the program of Jones (1966). Subroutine RKAM is a modification of subroutine RKAMSUB, which was written by G. J. Lastman and is available through the CDC CO-OP library (the CO-OP identification is D2 UTEX RKAMSUB). Subroutine GAUSEL was written by L. David Lewis, Space Environment Laboratory, National Oceanic and Atmospheric Administration. Subroutine FSW was written in conjunction with Helmut Kopka of the Max-Planck-Institut für Aeronomie, Lindau/Harz, Germany.

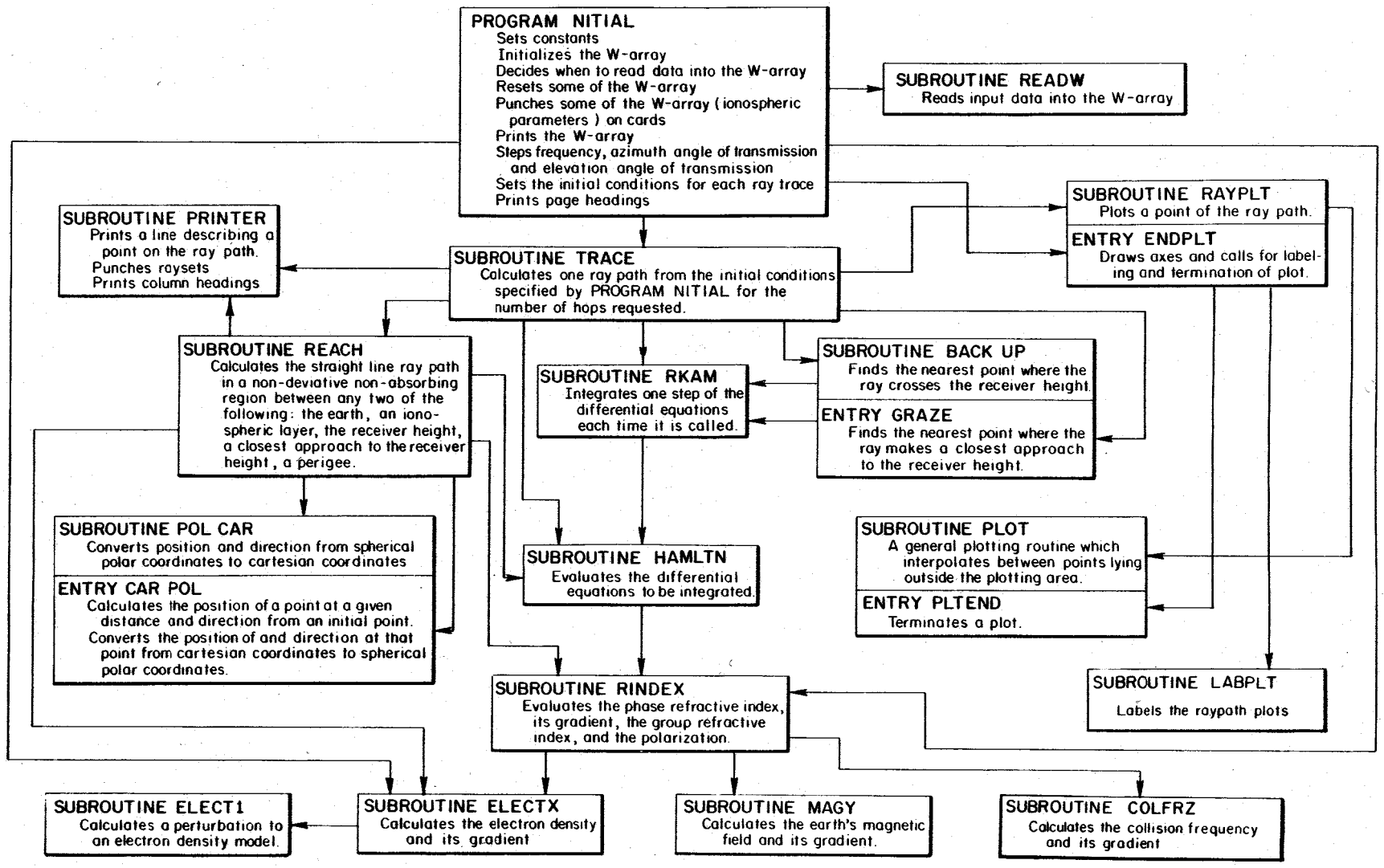


Figure 7. Block diagram for the ray tracing program.

Table 3. Definitions of the Parameters in Blank Common

Position in Common	Variable Name	Definition
1-20	R	The dependent variables in the differential equations being integrated--the definitions of the first six are fixed, but the others may be varied by the program user.
1	R(1)	r
2	R(2)	θ
3	R(3)	φ
4	R(4)	k_r
5	R(5)	k_θ
6	R(6)	k_φ
7-12	R(7)-R(12)	Those variables the user has chosen to integrate, taken in the following order: P -phase path in kilometers A -absorption in decibels Δf -Doppler shift in hertz s -geometrical path length in kilometers
13-20	R(13)-R(20)	Reserved for future expansion.
21	T	Group path in kilometers (the independent variable in the differential equations).
22	STP	Step length in group path.
23-42	DRDT	The derivatives of the dependent variables with respect to the independent variable T.

R and T are initialized in program NITIAL and changed in subroutines RKAM, REACH, and BACK UP.

STP is calculated in subroutine RKAM.

DRDT is calculated in subroutine HAMLTN and used in subroutine RKAM.

Table 4. Definitions of the Parameters in Common Block /CONST/

Position in Common	Variable Name	Definition
1	PI	π
2	PIT2	2π
3	PID2	$\pi/2$
4	DEGS	$180.0/\pi$
5	RAD	$\pi/180.0$
6	K	Ratio of the square of the plasma frequency to the electron density in $MHz^2 cm^3 = r_o c^2/\pi = e^2/(4\pi^2 \epsilon_o m)$, where r_o is the classical electron radius, c is the free space speed of light, e is the charge on the electron, m is the mass of the electron, and ϵ_o is the capacitivity of a vacuum.
7	C	Free space speed of light in km/sec.
8	LOGTEN	$\log_e 10$

These parameters are set in program NITIAL.

Table 5. Definitions of the Parameters in Common Block /RK/

Position in Common	Variable Name	Definition
1	N	The number of equations being integrated.
2	STEP	The initial step in group path in kilometers.
3	MODE	Defines type of integration used (same as W41), see Table 2.
4	E1MAX	Maximum allowable single step error (same as W42).
5	E1MIN	Minimum allowable single step error (= W42/W43).
6	E2MAX	Maximum step length (same as W45).
7	E2MIN	Minimum step length (same as W46).
8	FACT	Factor by which to increase or decrease step length (same as W47).
9	RSTART	Nonzero to initialize numerical integration, zero to continue integration.

These parameters are calculated in program NITIAL (some are temporarily reset in subroutine BACK UP) and are used in subroutine RKAM.

Table 6. Definition of the Parameters in Common Block /RIN/

Postion in Common	Variable Name	Definition
1, 2, 3	MODRIN	Description of version of RINDEX in BCD.
4	COLL	= 1 if this version of RINDEX includes collisions, = 0 otherwise.
5	FIELD	= 1 if this version of RINDEX includes the earth's magnetic field, = 0 otherwise.
6	SPACE	TRUE, if the ray is in a nondeviative, nonabsorbing medium.
7, 8	KAY2	k^2, square of the complex phase refractive index times ω^2/c^2.
9, 10	H	Hamiltonian (complex)
11, 12	PHPT	$\partial H/\partial t$ (complex)
13, 14	PHPR	$\partial H/\partial r$ (complex)
15, 16	PHPTH	$\partial H/\partial \theta$ (complex)
17, 18	PHPPH	$\partial H/\partial \varphi$ (complex)
19, 20	PHPOM	$\partial H/\partial \omega$ (complex)
21, 22	PHPKR	$\partial H/\partial k_r$ (complex)
23, 24	PHPKTH	$\partial H/\partial k_\theta$ (complex)
25, 26	PHPKPH	$\partial H/\partial k_\varphi$ (complex)
27, 28	KPHPK	$\vec{k} \cdot \partial H/\partial \vec{k}$ (complex) $= k_r \partial H/\partial k_r + k_\theta \partial H/\partial k_\theta + k_\varphi \partial H/\partial k_\varphi$
29, 30	POLAR	Characteristic polarization of the wave; equal to the ratio of the component of the electric field perpendicular with the earth's magnetic field to the transverse component of the electric field parallel with the earth's magnetic field (complex) (Budden, 1961, p. 49, eq. (5.13)).

Table 6. (Continued)

Position in Common	Variable Name	Definition
31, 32	LPOLAR	Characteristic longitudinal polarization of the wave; equal to the ratio of the longitudinal component of the electric field to the component of the electric field perpendicular with the earth's magnetic field. (complex) Budden, 1961, p. 54, eq. (5.38)).
33	SGN	= +1 or -1; used for ray tracing in complex space.

These parameters are calculated in subroutine RINDEX and used in subroutine HAMLTN.

Note: In some subroutines, the real and imaginary parts of the complex variables have separate names.

Table 7. Definitions of the Parameters in Common Block /FLG/ (See Subroutine TRACE)

Position in Common	Variable Name	Definition
1	NTYP	= 1 for extraordinary, = 2 for no field, = 3 for ordinary
2	NEWWR	Set equal to . TRUE. to tell subroutine RAYPLT there is a new W array.
3	NEWWP	Set equal to .TRUE. to tell subroutine PRINTR there is a new W array.
4	PENET	Set equal to .TRUE. if the ray just penetrated.
5	LINES	Number of lines printed on the current page.
6	IHOP	Hop number (at the beginning of each ray, subroutine TRACE sets this parameter to zero so that subroutine RAYPLT will begin a new line in plotting the ray path and subroutine PRINTR will print column headings and punch a transmitter rayset).
7	HPUNCH	The height to be punched on the raysets.

Table 8. Definitions of the Parameters in Common Block /XX/

Position in Common	Variable Name	Definition
1	MODX(1)	BCD name of the electron density subroutine.
2	MODX(2)	BCD name of the subroutine defining a perturbation to the electron density model.
3	X	X in Appleton-Hartree formula, square of the ratio of the plasma frequency to the wave frequency.
4	PXPR	$\frac{\partial X}{\partial r}$
5	PXPTH	$\frac{\partial X}{\partial \theta}$
6	PXPPH	$\frac{\partial X}{\partial \varphi}$
7	PXPT	$\frac{\partial X}{\partial t}$, where t is time; used for calculating Doppler shifts.
8	HMAX	Height of maximum electron density.

These parameters are calculated in subroutine ELECTX, possibly modified in subroutine ELECT1, and are mainly used in subroutine RINDEX.

Table 9. Definitions of the Parameters in Common Block /YY/

Position in Common	Variable Name	Definition
1	MODY	BCD name of the subroutine defining the earth's magnetic field.
2	Y	Y in the Appleton-Hartree formula, ratio of the electron gyrofrequency to the wave frequency.
3	PYPR	$\frac{\partial Y}{\partial r}$
4	PYPTH	$\frac{\partial Y}{\partial \theta}$
5	PYPPH	$\frac{\partial Y}{\partial \varphi}$
6	YR	Y_r, proportional to the component of the earth's magnetic field in the r direction.
7	PYRPR	$\frac{\partial Y_r}{\partial r}$
8	PYRPT	$\frac{\partial Y_r}{\partial \theta}$
9	PYRPP	$\frac{\partial Y_r}{\partial \varphi}$
10	YTH	Y_θ
11	PYTPR	$\frac{\partial Y_\theta}{\partial r}$
12	PYTPT	$\frac{\partial Y_\theta}{\partial \theta}$

Table 9. (Continued)

Position in Common	Variable Name	Definition
13	PYTPP	$\frac{\partial Y_\theta}{\partial \varphi}$
14	YPH	Y_φ
15	PYPPR	$\frac{\partial Y_\varphi}{\partial r}$
16	PYPPT	$\frac{\partial Y_\varphi}{\partial \theta}$
17	PYPPP	$\frac{\partial Y_\varphi}{\partial \varphi}$

These parameters are calculated in subroutine MAGY and are mainly used in subroutine RINDEX.

Table 10. Definitions of the Parameters in Common Block /ZZ/

Position in Common	Variable Name	Definition
1	MODZ	BCD name of the collision frequency subroutine.
2	Z	Z in the Appleton-Hartree formula, ratio of the electron-neutral collision frequency to the angular wave frequency.
3	PZPR	$\frac{\partial Z}{\partial r}$
4	PZPTH	$\frac{\partial Z}{\partial \theta}$
5	PZPPH	$\frac{\partial Z}{\partial \varphi}$

These parameters are calculated in subroutine COLFRZ and are mainly used in subroutine RINDEX.

Table 11. Definitions of the Parameters in Common Block /TRAC/

Position in Common	Variable Name	Definition
1	GROUND	.TRUE. if the ray is on the surface of the earth.
2	PERIGE	.TRUE. if the ray has just made a perigee.
3	THERE	.TRUE. if the ray is at the receiver height.
4	MINDIS	.TRUE. if the ray has just made a closest approach to the receiver height.
5	NEWRAY	Set equal to .TRUE. to tell subroutine REACH that this is a new ray.
6	SMT	An estimation of the vertical distance to an apogee or perigee of the ray.

These parameters are used for communication between subroutine TRACE and subroutines REACH and BACK UP.

Table 12. Definition of the Parameter in Common Block /COORD/

Position in Common	Variable Name	Definition
1	S	The straight line distance along the ray from the position of the ray where REACH was called to the present position.

This parameter is used for communication between subroutine REACH and subroutine POL CAR.

Table 13. Definitions of the Parameters in Common Block /PLT/

Position in Common	Variable Name	Definition
1	XMIN0, XL	The x coordinate of the left side of the plotting area in kilometers.
2	XMAX0, XR	The x coordinate of the right side of the plotting area in kilometers.
3	XMIN0, YB	The y coordinate of the bottom of the plotting area in kilometers.
4	YMAX0, YT	The y coordinate of the top of the plotting area in kilometers.
5	RESET	Set equal to one whenever the plotting area is changed.

These parameters are used for communication between subroutine RAYPLT and subroutine PLOT.

Table 14. Definitions of the Parameters in Common Block /DD/

Position in Common	Variable Name	Definition
1	IN	Intensity. IN = 0 specifies normal intensity. IN = 1 specifies high intensity.
2	IOR	Orientation. IOR = 0 specifies upright orientation. IOR = 1 specifies rotated orientation (90° counterclockwise).
3	IT	Italics (Font). IT = 0 specifies non-Italic (Roman) symbols. IT = 1 specifies Italic symbols.
4	IS	Symbol size. IS = 0 specifies miniature size. IS = 1 specifies small size. IS = 2 specifies medium size. IS = 3 specifies large size.
5	IC	Symbol case. IC = 0 specifies upper case. IC = 1 specifies lower case.
6	ICC	Character code, 0-63 (R1 format). ICC and IC together specify the symbol plotted.
7	IX	X-coordinate, 0-1023.
8	IY	Y-coordinate, 0-1023.

We also want to thank those who have used our program and have pointed out errors or made suggestions. In particular, we are grateful to Dr. T. M. Georges of the Wave Propagation Laboratory, National Oceanic and Atmospheric Administration, for his suggestions resulting from extensive use of the program, for development of some of the ionospheric models (DCHAPT, DTORUS, WAVE, WAVE2), and for financing part of the development of ray tracing through a spitze.

Examples of use of the ray tracing program are shown in the reports by Stephenson and Georges (1969) and Georges (1971).

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APPENDIX 1. LISTINGS OF THE MAIN PROGRAM AND SUBROUTINES IN THE MAIN DECK

This appendix contains listings of the main program and those subroutines that have only one version (with the exception of subroutine RAYPLT, which has a do-nothing version for users lacking a plotter to plot ray paths). Thus, the routines which form the contents of this appendix will be used in all ray path calculations.

Additionally, this appendix contains the main input parameter form for ray tracing and the input parameter forms for plotting. These forms are very useful when using the program because they indicate the input parameters needed for ray path calculations

The contents of this appendix are arranged as follows:

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INPUT PARAMETER FORM
FOR THREE-DIMENSIONAL RAY PATHS

Name _____ Project No. _____ Date _____

Ionospheric ID (3 characters) _____

Title (75 characters) _____

Models:	Electron density	_____	
	Perturbation	_____	
	Magnetic field	_____	
	Ordinary	_____	(W1 = + 1.)
	Extraordinary	_____	(W1 = - 1.)
	Collision frequency	_____	
Transmitter:	Height	_____	km, nautical miles, feet (W3)
	Latitude	_____	rad, deg, km (W4)
	Longitude	_____	rad, deg, km (W5)
	Frequency, initial	_____	MHz (W7)
	final	_____	(W8)
	step	_____	(W9)
	Azimuth angle, initial	_____	rad, deg clockwise of north (W11)
	final	_____	(W12)
	step	_____	(W13)
	Elevation angle, initial	_____	rad, deg (W15)
	final	_____	(W16)
	step	_____	(W17)
Receiver:	Height	_____	km, nautical miles, feet (W20)
Penetrating rays:	Wanted	_____	(W21 = 0.)
	Not wanted	_____	(W21 = 1.)
Maximum number of hops		_____	(W22)
Maximum number of steps per hop		_____	(W23)
Maximum allowable error per step		_____	(W42)
Additional calculations:			= 1. to integrate = 2. to integrate and print
	Phase path	_____	(W57)
	Absorption	_____	(W58)
	Doppler shift	_____	(W59)
	Path length	_____	(W60)
	Other	_____	
		_____	
		_____	

Printout: Every _____ steps of the ray trace (W71)

Punched cards (raysets): _____ (W72 = 1.)

```
      PROGRAM NITIAL                                                    NITI001
C          SETS THE INITIAL CONDITIONS FOR EACH RAY AND CALLS TRACE     NITI002
      DIMENSION MFLD(2)                                                 NITI003
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RAD,K,C,LOGTEN                   NITI004
      COMMON /FLG/ NTYP,NEWWR,NEWWP,PENET,LINES,IHOP,HPUNCH             NITI005
      COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,N2,N2I,PNP(10),POLAR,     NITI006
     1             LPOLAR,SGN                                           NITI007
      COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART       NITI008
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  NITI009
      COMMON /YY/ MODY,Y(16) /ZZ/ MODZ,Z(4)                             NITI010
      COMMON R(20),T,STP,OROT(20) /WW/ ID(10),W0,W(400)                 NITI011
      EQUIVALENCE (RAY,W(1)),(EARTHR,W(2)),(XMTRH,W(3)),(TLAT,W(4)),    NITI012
     1 (TLON,W(5)),(F,W(6)),(FBEG,W(7)),(FEND,W(8)),(FSTEP,W(9)),       NITI013
     2 (AZ1,W(10)),(AZBEG,W(11)),(AZEND,W(12)),(AZSTEP,W(13)),          NITI014
     3 (BETA,W(14)),(ELBEG,W(15)),(ELEND,W(16)),(ELSTEP,W(17)),         NITI015
     4 (ONLY,W(21)),(HOP,W(22)),(MAXSTP,W(23)),(PLAT,W(24)),(PLON,W(25))NITI016
     5,(INTYP,W(41)),(MAXERR,W(42)),(ERATIO,W(43)),(STEP1,W(44)),       NITI017
     6 (STPMAX,W(45)),(STPMIN,W(46)),(FACTR,W(47)),(SKIP,W(71)),        NITI018
     7 (RAYSET,W(72)),(PLT,W(81)),(PERT,W(150))                         NITI019
      LOGICAL SPACE,NEWWR,NEWWP,PENET                                   NITI020
      REAL N2,N2I,LOGTEN,K,MAXSTP,INTYP,MAXERR,MU                       NITI021
      COMPLEX PNP,POLAR,LPOLAR                                          NITI022
      NDATE=IDATE(0)                                                    NITI025
      SECOND=KLOCK(0)*.001                                              NITI026
      KOLL=4H  NO                                                       NITI027
      IF (COLL.NE.0.) KOLL=4HWITH                                       NITI028
C********* CONSTANTS                                                    NITI029
      PI=3.1415926536                                                   NITI030
      PIT2=2.*PI                                                        NITI031
      PID2=PI/2.                                                        NITI032
      DEGS=180./PI                                                      NITI033
      RAD=PI/180.                                                       NITI034
      C=2.997925E5                                                      NITI035
      K=2.81785E-15*C**2/PI                                             NITI036
      LOGTEN=ALOG(10.)                                                  NITI037
C********* INITIALIZE SOME VARIABLES IN THE W ARRAY                     NITI038
      DO 5 NW=1,400                                                     NITI039
    5 W(NW)=0.                                                          NITI040
      PLON=0.                                                           NITI041
      PLAT=PID2                                                         NITI042
      EARTHR=6370.                                                      NITI043
      INTYP=3.                                                          NITI044
      MAXERR=1.E-4                                                      NITI045
      ERATIO=50.                                                        NITI046
      STEP1=1.                                                          NITI047
      STPMAX=100.                                                       NITI048
      STPMIN=1.E-8                                                      NITI049
      FACTR=0.5                                                         NITI050
      MAXSTP=1000.                                                      NITI051
      HOP=1.                                                            NITI052
C********* READ W ARRAY AND PRINT NON-ZERO VALUES                       NITI053
   10 CALL READ W                                                       NITI054
      F=BETA=AZ1=0.                                                     NITI055
      IF (SKIP.EQ.0.) SKIP=MAXSTP                                       NITI056
   12 RAY=SIGN(1.,RAY)                                                  NITI057
      NTYP=2.+FIELD*RAY                                                 NITI058
      GO TO (13,14,15), NTYP                                            NITI059
   13 MFLD(1)=8HEXTRAORD                                                NITI060
      MFLD(2)=5HINARY                                                   NITI061
      GO TO 16                                                          NITI062
   14 MFLD(1)=8HNO FIELD                                                NITI063
      MFLD(2)=1H                                                        NITI064
      GO TO 16                                                          NITI065
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   15 MFLD(1)=8HORDINARY                                                NITI066
      MFLD(2)=8H                                                        NITI067
   16 MODSAV=MODX(2)                                                    NITI068
      IF (PERT.EQ.0.) MODX(2)=6H                                        NITI069
      IF (RAYSET.NE.0.) PUNCH 2000, ID,MODX(1),(W(NW),NW=101,107),      NITI070
     1 MODX(2),(W(NW),NW=151,157),MODY,(W(NW),NW=201,207),              NITI071
     2 MODZ,(W(NW),NW=251,257)                                          NITI072
 2000 FORMAT (10A8,4(/A8,2X,7E10.3))                                    NITI073
      PRINT 1000, ID,NDATE,MODX,MODY,MODZ,MODRIN,MFLD,KOLL              NITI074
 1000 FORMAT (1H1,10A8,25X,A8/4(1X,A8),24X,3A8,1X,A6,A5,1X,A4,          NITI075
     1 11H COLLISIONS/)                                                 NITI076
      PRINT 1050                                                        NITI077
 1050 FORMAT (85H INITIAL VALUES FOR THE W ARRAY -- ALL ANGLES IN RADIANNITI078
     1S, ONLY NONZERO VALUES PRINTED/)                                  NITI079
      DO 17 NW=1,400                                                    NITI080
      IF (W(NW).NE.0.) PRINT 1700, NW,W(NW)                             NITI081
 1700 FORMAT (I4,E19.11)                                                NITI082
   17 CONTINUE                                                          NITI083
C********* LET SUBROUTINES PRINTR AND RAYPLT KNOW THERE IS A NEW W ARRAYNITI084
      NEWWP=.TRUE.                                                      NITI085
      NEWWR=.TRUE.                                                      NITI086
C********* INITIALIZE PARAMETERS FOR INTEGRATION SUBROUTINE RKAM        NITI087
      N=6                                                               NITI088
      DO 20 NR=7,20                                                     NITI089
      IF (W(50+NR).NE.0.) N=N+1                                         NITI090
   20 CONTINUE                                                          NITI091
      MODE=INTYP                                                        NITI092
      STEP=STEP1                                                        NITI093
      E1MAX=MAXERR                                                      NITI094
      E1MIN=MAXERR/ERATIO                                               NITI095
      E2MAX=STPMAX                                                      NITI096
      E2MIN=STPMIN                                                      NITI097
      FACT=FACTR                                                        NITI098
C********* DETERMINE TRANSMITTER LOCATION IN COMPUTATIONAL COORDINATE   NITI099
C********* SYSTEM (GEOMAGNETIC COORDINATES IF DIPOLE FIELD IS USED)     NITI100
      R0=EARTHR+XMTRH                                                   NITI101
      SP=SIN (PLAT)                                                     NITI102
      CP=SIN (PID2-PLAT)                                                NITI103
      SDPH=SIN (TLON-PLON)                                              NITI104
      CDPH=SIN (PID2-(TLON-PLON))                                       NITI105
      SL=SIN (TLAT)                                                     NITI106
      CL=SIN (PID2-TLAT)                                                NITI107
      ALPHA=ATAN2(-SDPH*CP,-CDPH*CP*SL+SP*CL)                           NITI108
      TH0=ACOS (CDPH*CP*CL+SP*SL)                                       NITI109
      PH0=ATAN2(SDPH*CL,CDPH*SP*CL-CP*SL)                               NITI110
C********* LOOP ON FREQUENCY, AZIMUTH ANGLE, AND ELEVATION ANGLE        NITI111
      NFREQ=1                                                           NITI112
      IF (FSTEP.NE.0.) NFREQ=(FEND-FBEG)/FSTEP+1.5                      NITI113
      NAZ=1                                                             NITI114
      IF (AZSTEP.NE.0.) NAZ=(AZEND-AZBEG)/AZSTEP+1.5                    NITI115
      NBETA=1                                                           NITI116
      IF (ELSTEP.NE.0.) NBETA=(ELEND-ELBEG)/ELSTEP+1.5                  NITI117
      DO 50 NF=1,NFREQ                                                  NITI118
      F=FBEG+(NF-1)*FSTEP                                               NITI119
      DO 45 J=1,NAZ                                                     NITI120
      AZ1=AZBEG+(J-1)*AZSTEP                                            NITI121
      AZA=AZ1*DEGS                                                      NITI122
      GAMMA=PI-AZ1+ALPHA                                                NITI123
      SGAMMA=SIN (GAMMA)                                                NITI124
      CGAMMA=SIN (PID2-GAMMA)                                           NITI125
      DO 40 I=1,NBETA                                                   NITI126
      BETA=ELBEG+(I-1)*ELSTEP                                           NITI127
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      EL=BETA*DEGS                                                      NITI128
      CBETA=SIN (PID2-BETA)                                             NITI129
      R(1)=R0                                                           NITI130
      R(2)=TH0                                                          NITI131
      R(3)=PH0                                                          NITI132
      R(4)=SIN (BETA)                                                   NITI133
      R(5)=CBETA*CGAMMA                                                 NITI134
      R(6)=CBETA*SGAMMA                                                 NITI135
      T=0.                                                              NITI136
      RSTART=1.                                                         NITI137
C     SGN=1.  (NEED FOR RAY TRACING IN COMPLEX SPACE.)                  NITI138
C********* ALLOW IONOSPHERIC MODEL SUBROUTINES TO READ AND PRINT DATA   NITI139
      CALL RINDEX                                                       NITI140
      IF (I.NE.1.AND.NPAGE.LT.3.AND.LINES.LE.17) GO TO 25               NITI141
      NPAGE=LINES=0                                                     NITI142
      PRINT 1000, ID,NDATE,MODX,MODY,MODZ,MODRIN,MFLD,KOLL              NITI143
      PRINT 2400, F,AZA                                                 NITI144
 2400 FORMAT (18X,11HFREQUENCY =,F12.6,37H MHZ, AZIMUTH ANGLE OF TRANSMINITI145
     1SSION =,F12.6,4H DEG)                                             NITI146
   25 NPAGE=NPAGE+1                                                     NITI147
      PRINT 2500, EL                                                    NITI148
 2500 FORMAT (/31X,33HELEVATION ANGLE OF TRANSMISSION =,F12.6,4H DEG/)  NITI149
      IF (N2.GT.0.) GO TO 30                                            NITI150
      CALL ELECTX                                                       NITI151
      FN=SIGN (SQRT (ABS (X))*F,X)                                      NITI152
      PRINT 2900, FN                                                    NITI153
 2900 FORMAT (58H0TRANSMITTER IN EVANESCENT REGION, TRANSMISSION IMPOSSINITI154
     1BLE/20H0PLASMA FREQUENCY = ,E17.10)                               NITI155
      GO TO 44                                                          NITI156
   30 MU=SQRT (N2/(R(4)**2+R(5)**2+R(6)**2))                            NITI157
      DO 34 NN=4,6                                                      NITI158
   34 R(NN)=R(NN)*MU                                                    NITI159
      DO 35 NN=7,N                                                      NITI160
   35 R(NN)=0.                                                          NITI161
      CALL TRACE                                                        NITI162
      OSEC=SECOND                                                       NITI163
      SECOND=KLOCK(0)*.001                                              NITI164
      DIFF=SECOND-OSEC                                                  NITI165
      PRINT 3500, DIFF                                                  NITI166
 3500 FORMAT (36X,26HTHIS RAY CALCULATION TOOK ,F8.3,4H SEC)            NITI167
      IF (PENET.AND.ONLY.NE.0..AND.IHOP.EQ.1) GO TO 44                  NITI168
   40 CONTINUE                                                          NITI169
   44 IF (PLT.NE.0.) CALL ENDPLT                                        NITI170
   45 CONTINUE                                                          NITI171
      IF(PENET.AND.ONLY.NE.0..AND.IHOP.EQ.1.AND.NAZ.EQ.1.AND.NBETA.EQ.1)NITI172
     1 GO TO 55                                                         NITI173
   50 CONTINUE                                                          NITI174
   55 IF (RAYSET.NE.0.) PUNCH 5000                                      NITI175
 5000 FORMAT (78X,1H-)                                                  NITI176
      MODX(2)=MODSAV                                                    NITI177
      GO TO 10                                                          NITI178
         END                                                            NITI179-
```

```
      SUBROUTINE READ W                                                 READ001
C                          READS W ARRAY                                READ002
C       A 1 IN THE FOLLOWING COLUMNS WILL MAKE THE DESCRIBED CONVERSIONSREAD003
COL 18   DEGREES TO RADIANS                                             READ004
COL 19   GREAT CIRCLE DISTANCE IN KM TO RADIANS                         READ005
COL 20   NAUTICAL MILES TO KM                                           READ006
COL 21   FEET TO KM                                                     READ007
C                                                                       READ008
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RAD,DUM(3)                       READ009
      COMMON /WW/ ID(10),W0,W(400)                                      READ010
      EQUIVALENCE (EARTHR,W(2))                                         READ011
      INTEGER DEG,FEET                                                  READ012
      READ 1000, ID                                                     READ013
 1000 FORMAT (10A8)                                                     READ014
      IF (EOF,60) 3,4                                                   READ015
    3 CALL EXIT                                                         READ016
    4 READ 1100, NW,W(NW),DEG,KM,NM,FEET                                READ017
 1100 FORMAT (I3,E14.7,5I1)                                             READ018
      IF (NW.EQ.0) GO TO 10                                             READ019
      IF (NW.GT.0.AND.NW.LE.400) GO TO 5                                READ020
      PRINT 4000, NW                                                    READ021
 4000 FORMAT(15H1THE SUBSCRIPT ,I3,77H ON THE W-ARRAY INPUT IS OUT OF BOREAD022
     1UNDS.  ALLOWABLE VALUES ARE 1 THROUGH 400. )                      READ023
      CALL EXIT                                                         READ024
    5 IF (DEG.NE.0.) W(NW)=W(NW)*RAD                                    READ025
      IF (KM.NE.0) W(NW)=W(NW)/EARTHR                                   READ026
      IF (NM.NE.0) W(NW)=W(NW)*1.852                                    READ027
      IF (FEET.NE.0) W(NW)=W(NW)*3.048006096E-4                         READ028
      GO TO 4                                                           READ029
   10 RETURN                                                            READ030
         END                                                            READ031-
```

```
      SUBROUTINE TRACE                                                  TRAC001
C          CALCULATES THE RAY PATH                                      TRAC002
      DIMENSION ROLD(20),DROLD(20)                                      TRAC003
      COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART       TRAC004
      COMMON /FLG/ NTYP,NEWWR,NEWWP,PENET,LINES,IHOP,HPUNCH             TRAC005
      COMMON /TRAC/ GROUND,PERIGE,THERE,MINDIS,NEWRAY,SMT               TRAC006
      COMMON /RIN/ MOORIN(3),COLL,FIELD,SPACE,N2,PNP(10),POLAR,LPOLAR   TRAC007
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  TRAC008
      COMMON R(20),T,STP,DRDT(20) /WW/ ID(10),W0,W(400)                 TRAC009
      LOGICAL SPACE,HOME,WASNT,UNDRGD,GROUND,PERIGE,THERE,MINDIS,NEWWR, TRAC010
     1        NEWWP,PENET,NEWRAY,WAS                                    TRAC011
      REAL MAXSTP                                                       TRAC012
      COMPLEX N2,PNP,POLAR,LPOLAR                                       TRAC013
      EQUIVALENCE (EARTHR,W(2)),(RCVRH,W(20)),(HOP,W(22)),(MAXSTP,W(23))TRAC014
     1,(SKIP,W(71)),(RAYSET,W(72)),(PLT,W(81))                          TRAC015
      NHOP=HOP                                                          TRAC016
      MAX=MAXSTP                                                        TRAC017
      NSKIP=SKIP                                                        TRAC018
      RSTART=1.                                                         TRAC019
      CALL HAMLTN                                                       TRAC020
      HOME=DRDT(1)*(R(1)-EARTHR-RCVRH).GE.0.                            TRAC021
C********* IHOP=0 TELLS PRINTR TO PRINT HEADING AND PUNCH A TRANSMITTER TRAC022
C********* RAYSET AND TELLS RAYPLT TO START A NEW RAY                  TRAC023
      IHOP=0                                                            TRAC024
      CALL PRINTR (8HXMTR    ,0.)                                       TRAC025
      IF (PLT.NE.0.) CALL RAYPLT                                        TRAC026
      HTMAX=0.                                                          TRAC027
      NEWRAY=.TRUE.                                                     TRAC028
      THERE=R(1)-EARTHR.EQ.RCVRH                                        TRAC029
```

```
C********* LOOP ON NUMBER OF HOPS                                          TRAC030
   10 IHOP=IHOP+1                                                          TRAC031
      IF (IHOP.GT.NHOP) RETURN                                             TRAC032
      PENET=.FALSE.                                                        TRAC033
      APHT=RCVRH                                                           TRAC034
C********* LOOP ON MAXIMUM NUMBER OF STEPS PER HOP                         TRAC035
      DO 79 J=1,MAX                                                        TRAC036
      H=R(1)-EARTHR                                                        TRAC037
      IF (ABS(H-RCVRH).GT.ABS(APHT-RCVRH)) APHT=H                          TRAC038
      HTMAX=AMAX1(H,HTMAX)                                                 TRAC039
      IF (.NOT.SPACE) GO TO 12                                             TRAC040
      CALL REACH                                                           TRAC041
      RSTART=1.                                                            TRAC042
      H=R(1)-EARTHR                                                        TRAC043
      IF (ABS(H-RCVRH).GT.ABS(APHT-RCVRH)) APHT=H                          TRAC044
      HTMAX=AMAX1(H,HTMAX)                                                 TRAC045
      IF (.NOT.SPACE) GO TO 12                                             TRAC046
      IF (PERIGE) CALL PRINTR (8HPERIGEE ,0.)                              TRAC047
      IF (THERE) GO TO 51                                                  TRAC048
      IF (MINDIS) GO TO 40                                                 TRAC049
      IF (GROUND) GO TO 60                                                 TRAC050
      IF (PLT.NE.0.) CALL RAYPLT                                           TRAC051
      IF (PERIGE) GO TO 79                                                 TRAC052
   12 DO 13 L=1,N                                                          TRAC053
      ROLD(L)=R(L)                                                         TRAC054
   13 DROLD(L)=DRDT(L)                                                     TRAC055
      TOLD=T                                                               TRAC056
      WAS=THERE                                                            TRAC057
      CALL RKAM                                                            TRAC058
      H=R(1)-EARTHR                                                        TRAC059
      THERE=.FALSE.                                                        TRAC060
      WASNT=.NOT.HOME                                                      TRAC061
      HOME=DRDT(1)*(H-RCVRH).GE.0.                                         TRAC062
      TMP=(DRDT(1)-DROLD(1))*(T-TOLD)                                      TRAC063
      SMT=0.                                                               TRAC064
      IF (TMP.NE.0.) SMT=0.5*(R(1)-ROLD(1)+0.5*TMP)**2/ABS(TMP)            TRAC065
      IF (((H-RCVRH)*(ROLD(1)-EARTHR-RCVRH).LT.0..AND..NOT.WAS).OR.        TRAC066
     1 (WAS.AND.DRDT(1)*DROLD(1).LT.0..AND.HOME)) GO TO 50                 TRAC067
      IF (HOME.AND.WASNT) GO TO 30                                         TRAC068
      IF (H.LT.0..OR.DRDT(1).GT.0..AND.DROLD(1).LT.0..AND.SMT.GT.H)        TRAC069
     1 GO TO 20                                                            TRAC070
      IF (DROLD(1).LT.0..AND.DRDT(1).GT.0.) CALL PRINTR(8HPERIGEE ,0.)     TRAC071
      IF (DROLD(1).GT.0..AND.DRDT(1).LT.0.) CALL PRINTR(8HAPOGEE  ,0.)     TRAC072
      IF (DROLD(2)*DRDT(2).LT.0.) CALL PRINTR(8HMAX LAT ,0.)               TRAC073
      IF (DROLD(3)*DRDT(3).LT.0.) CALL PRINTR(8HMAX LONG,0.)               TRAC074
      DO 14 I=4,6                                                          TRAC075
      IF (ROLD(I)*R(I).LT.0.) CALL PRINTR(8HWAVE REV,0.)                   TRAC076
   14 CONTINUE                                                             TRAC077
      GO TO 75                                                             TRAC078
C********* RAY WENT UNDERGROUND                                            TRAC079
   20 CALL BACK UP(0.)                                                     TRAC080
      GO TO 60                                                             TRAC081
C********* RAY MAY HAVE MADE A CLOSEST APPROACH                            TRAC082
   30 CALL GRAZE(RCVRH)                                                    TRAC083
      IF (THERE) GO TO 51                                                  TRAC084
   40 DRDT(1)=0.                                                           TRAC085
      HPUNCH=R(1)-EARTHR                                                   TRAC086
      CALL PRINTR(8HMIN DIST,RAYSET)                                       TRAC087
      IF (PLT.NE.0.) CALL RAYPLT                                           TRAC088
      IF (IHOP.GE.NHOP) RETURN                                             TRAC089
      IHOP=IHOP+1                                                          TRAC090
      CALL PRINTR (8HMIN DIST,RAYSET)                                      TRAC091
      GO TO 89                                                             TRAC092
```

```
C********* RAY CROSSED RECEIVER HEIGHT                                    TRAC093
   50 CALL BACK UP(RCVRH)                                                 TRAC094
      THERE=.TRUE.                                                        TRAC095
   51 R(1)=EARTHR+RCVRH                                                   TRAC096
      HTMAX=AMAX1(RCVRH,HTMAX)                                            TRAC097
      HPUNCH=APHT                                                         TRAC098
      CALL PRINTR(8HRCVR    ,RAYSET)                                      TRAC099
      IF (PLT.NE.0.) CALL RAYPLT                                          TRAC100
      IF (RCVRH.NE.0.) GO TO 89                                           TRAC101
      IF (IHOP.GE.NHOP) RETURN                                            TRAC102
      IHOP=IHOP+1                                                         TRAC103
      APHT=RCVRH                                                          TRAC104
C********* GROUND REFLECT                                                 TRAC105
   60 R(1)=EARTHR                                                         TRAC106
      IF (ABS(RCVRH).GT.ABS(APHT-RCVRH)) APHT=0.                          TRAC107
      R(4)=ABS (R(4))                                                     TRAC108
      DRDT(1)=ABS (DRDT(1))                                               TRAC109
      RSTART=1.                                                           TRAC110
      HPUNCH=HTMAX                                                        TRAC111
      CALL PRINTR(8HGRND REF,RAYSET)                                      TRAC112
      HTMAX=0.                                                            TRAC113
      IF (RCVRH.NE.0.) GO TO 65                                           TRAC114
      THERE=.TRUE.                                                        TRAC115
      HPUNCH=APHT                                                         TRAC116
      CALL PRINTR (8HRCVR    ,RAYSET)                                     TRAC117
      GO TO 89                                                            TRAC118
   65 H=0.                                                                TRAC119
      THERE=.FALSE.                                                       TRAC120
C*********                                                                TRAC121
   75 IF (PLT.NE.0.) CALL RAYPLT                                          TRAC122
      IF (H.GT.HMAX.AND.H.GT.RCVRH.AND.DRDT(1).GT.0.) GO TO 90            TRAC123
      IF (MOD(J,NSKIP).EQ.0) CALL PRINTR(8H        ,0.)                   TRAC124
   79 CONTINUE                                                            TRAC125
C********* EXCEEDED MAXIMUM NUMBER OF STEPS                               TRAC126
      HPUNCH=H                                                            TRAC127
      CALL PRINTR(8HSTEP MAX,RAYSET)                                      TRAC128
      RETURN                                                              TRAC129
C*********                                                                TRAC130
   89 HOME=.TRUE.                                                         TRAC131
      GO TO 10                                                            TRAC132
C********* RAY PENETRATED                                                 TRAC133
   90 PENET=.TRUE.                                                        TRAC134
      HPUNCH=H                                                            TRAC135
      CALL PRINTR(8HPENETRAT,RAYSET)                                      TRAC136
      RETURN                                                              TRAC137
         END                                                              TRAC138
```

```
      SUBROUTINE BACK UP(HS)                                              BACK001
      COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART         BACK002
      COMMON /TRAC/ GROUND,PERIGE,THERE,MINDIS,NEWRAY,SMT                 BACK003
      COMMON R(20),T,STP,DRDT(20) /WW/ ID(10),W0,W(400)                   BACK004
      EQUIVALENCE (EARTHR,W(2)),(INTYP,W(41)),(STEP1,W(44))               BACK005
      REAL INTYP                                                          BACK006
      LOGICAL GROUND,PERIGE,THERE,MINDIS,NEWRAY,HOME                      BACK007
C********* DIAGNOSTIC PRINTOUT                                            BACK008
C        CALL PRINTR (8HBACK UP0,0.)                                      BACK009
C********* GOING AWAY FROM THE HEIGHT HS                                  BACK010
      HOME=DRDT(1)*(R(1)-EARTHR-HS).GE.0.                                 BACK011
      IF (HS.GT.0..AND..NOT.HOME.OR.HS.EQ.0..AND.DRDT(1).GT.0.) GO TO 30BACK012
```

```
C********* FIND NEAREST INTERSECTION OF RAY WITH THE HEIGHT HS
      DO 10 I=1,10
      STEP=-(R(1)-EARTHR-HS)/DRDT(1)
      STEP=SIGN(AMIN1(ABS(STP),ABS(STEP)),STEP)
      IF (ABS(R(1)-EARTHR-HS).LT..5E-4.AND.ABS(STEP).LT.1.) GO TO 60
C********* DIAGNOSTIC PRINTOUT
C         CALL PRINTR(8HBACK UP1,0.)
      MODE=1
      RSTART=1.
      CALL RKAM
   10 RSTART=1.
C
C********* FIND NEAREST CLOSEST APPROACH OF RAY TO THE HEIGHT HS
      ENTRY GRAZE
      THERE=.FALSE.
C********* DIAGNOSTIC PRINTOUT
C         CALL PRINTR (8HGRAZE 0 ,0.)
      IF (SMT.GT.ABS(R(1)-EARTHR-HS)) GO TO 30
      DO 20 I=1,10
      STEP=-R(4)/DRDT(4)
      STEP=SIGN(AMIN1(ABS(STP),ABS(STEP)),STEP)
      IF (ABS(R(4)).LE.1.E-6.AND.ABS(STEP).LT.1.) GO TO 60
C********* DIAGNOSTIC PRINTOUT
C         CALL PRINTR (8HGRAZE 1 ,0.)
      MODE=1
      RSTART=1.
      CALL RKAM
      RSTART=1.
      IF (DRDT(4)*(R(1)-EARTHR-HS).LT.0.) GO TO 30
      IF(R(5).EQ.0..AND.R(6).EQ.0.) GO TO 60
   20 CONTINUE
C********* IF A CLOSEST APPROACH COULD NOT BE FOUND IN 10 STEPS, IT
C********* PROBABLY MEANS THAT THE RAY INTERSECTS THE HEIGHT HS
   30 CONTINUE
C********* DIAGNOSTIC PRINTOUT
C         CALL PRINTR (8HBACK UP2,0.)
      MODE=1
C********* ESTIMATE DISTANCE TO NEAREST INTERSECTION OF RAY WITH HEIGHT
C********* HS BEHIND THE PRESENT RAY POINT
      STEP=(-R(4)-SQRT(R(4)**2-2.*(R(1)-EARTHR-HS)*DRDT(4)))/DRDT(4)
      RSTART=1.
      CALL RKAM
      RSTART=1.
C********* FIND NEAREST INTERSECTION OF RAY WITH HEIGHT HS
      DO 40 I=1,10
      STEP=-(R(1)-EARTHR-HS)/DRDT(1)
      STEP=SIGN(AMIN1(ABS(STP),ABS(STEP)),STEP)
      IF (ABS(R(1)-EARTHR-HS).LT..5E-4.AND.ABS(STEP).LT.1.) GO TO 60
C********* DIAGNOSTIC PRINTOUT
C         CALL PRINTR (8HBACK UP3,0.)
      MODE=1
      RSTART=1.
      CALL RKAM
   40 RSTART=1.
   50 THERE=.TRUE.
C********* RESET STANDARD MODE AND INTEGRATION TYPE
   60 MODE=INTYP
      STEP=STEP1
      RETURN
         END
```

```
      SUBROUTINE REACH                                                  REAC001
C          CALCULATES THE STRAIGHT LINE RAY PATH BETWEEN THE EARTH      REAC002
C          AND THE IONOSPHERE OR BETWEEN IONOSPHERIC LAYERS             REAC003
      COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART       REAC004
      COMMON /TRAC/ GROUND,PERIGE,THERE,MINDIS,NEWRAY,SMT               REAC005
      COMMON /COORD/ S                                                  REAC006
      COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,N2,N2I,PNP(10),POLAR,     REAC007
     1             LPOLAR                                               REAC008
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  REAC009
      COMMON P(20),T,STP,DRDT(20) /WW/ ID(10),WD,W(400)                 REAC010
      EQUIVALENCE (EARTHR,W(2)),(XMTRH,W(3)),(RCVRH,W(20))              REAC011
      LOGICAL CROSS,CROSSG,CROSSR,SPACE,GROUND,PERIGE,THERE,MINDIS,     REAC012
     1        NEWRAY,RSPACE                                             REAC013
      REAL N2,N2I                                                       REAC 14
      COMPLEX PNP,POLAR,LPOLAR                                          REAC 15
      DATA (NSTEP=500)                                                  REAC 16
      CALL HAMLTN                                                       REAC 17
      H=R(1)-EARTHR                                                     REAC 18
      IF (.NOT.NEWRAY.AND..NOT.RSPACE) CALL PRINTR(8HEXIT ION,0.)       REAC 19
      NEWRAY=.FALSE.                                                    REAC 20
      V=SQRT(R(4)**2+R(5)**2+R(6)**2)                                   REAC 21
C********* NORMALIZE THE WAVE NORMAL DIRECTION TO ONE                   REAC 22
      R(4)=R(4)/V                                                       REAC 23
      R(5)=R(5)/V                                                       REAC 24
      R(6)=R(6)/V                                                       REAC 25
C********* NEGATIVE OF DISTANCE ALONG RAY TO CLOSEST APPROACH TO CENTER REAC 26
C********* OF EARTH                                                     REAC 27
      UP=R(1)*R(4)                                                      REAC 28
      RADG=EARTHR**2-R(1)**2*(R(5)**2+R(6)**2)                          REAC 29
      DISTG=SQRT (AMAX1(0.,RADG))                                       REAC 30
C********* DISTANCE ALONG RAY TO FIRST INTERSECTION WITH OR CLOSEST     REAC 31
C********* APPROACH TO THE EARTH                                        REAC 32
      SG=-UP-DISTG                                                      REAC 33
C********* CROSSG IS TRUE IF THE RAY WILL INTERSECT OR TOUCH THE EARTH  REAC 34
      CROSSG=UP.LT.0..AND.RADG.GE.0.                                    REAC 35
      RADR=(EARTHR+RCVRH)**2-R(1)**2*(R(5)**2+R(6)**2)                  REAC 36
      DISTR=SQRT (AMAX1(0.,RADR))                                       REAC 37
C********* DISTANCE ALONG RAY TO THE FIRST INTERSECTION WITH OR CLOSEST REAC 38
C********* APPROACH TO THE RECEIVER HEIGHT                              REAC 39
      SR=DISTR-UP                                                       REAC 40
      IF (UP.LT.0..AND.DISTR.LT.-UP.AND.R(1).NE.EARTHR+RCVRH) SR=-DISTR REAC 41
     1 -UP                                                              REAC 42
C********* CROSSR IS TRUE IF THE RAY WILL INTERSECT WITH OR MAKE A      REAC 43
C********* CLOSEST APPROACH TO THE RECEIVER HEIGHT                      REAC 44
      CROSSR=R(4).LT.0..OR.R(1).LT.(EARTHR+RCVRH)                       REAC 45
      CROSS=CROSSG.OR.CROSSR                                            REAC 46
C********* MAXIMUM DISTANCE IN WHICH TO LOOK FOR THE IONOSPHERE         REAC 47
      S1=AMIN1(SR,SG)                                                   REAC 48
      IF(.NOT.CROSSG) S1=SR                                             REAC 49
      IF (UP.GE.0.) GO TO 15                                            REAC 50
      CROSS=.TRUE.                                                      REAC 51
C********* IF RAY IS GOING DOWN, S1 IS AT MOST THE DISTANCE TO A PERIGEEREAC 52
      S1=AMIN1(S1,-UP)                                                  REAC 53
C********* CONVERT THE POSITION AND DIRECTION OF THE RAY TO CARTESIAN   REAC 54
C********* COORDINATES                                                  REAC 55
   15 CALL POL CAR                                                      REAC 56
      SSTEP=100.                                                        REAC 57
      S=SSTEP                                                           REAC 58
      DO 20 I=1,NSTEP                                                   REAC 59
      IF ((S-SSTEP).GT.S1.AND.CROSS) GO TO 25                           REAC 60
C********* CONVERT POSITION AND DIRECTION TO SPHERICAL POLAR COORDINATESREAC 61
C********* AT A DISTANCE S ALONG THE RAY                                REAC 62
      CALL CAR POL                                                      REAC 63
      CALL ELECTX                                                       REAC 64
C********* FREE SPACE                                                   REAC 65
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      IF (X.EQ.0.) GO TO 20                                             REAC 66
      CALL RINDEX                                                       REAC 67
C********* EFFECTIVELY FREE SPACE                                       REAC 68
      IF (SPACE) GO TO 20                                               REAC 69
      IF (SSTEP.LT.0.5E-4) GO TO 25                                     REAC 70
C********* RAY IN THE IONOSPHERE.  STEP BACK OUT                        REAC 71
      S=S-SSTEP                                                         REAC 72
C********* DECREASE STEP SIZE                                           REAC 73
      SSTEP=SSTEP/10.                                                   REAC 74
   20 S=S+SSTEP                                                         REAC 75
      PRINT 2000, NSTEP                                                 REAC 76
 2000 FORMAT (9H EXCEEDED,I5,26H STEPS IN SUBROUTINE REACH)             REAC 77
      CALL EXIT                                                         REAC 78
   25 IF(CROSS) S=AMIN1(S,S1)                                           REAC 79
C********* CONVERT POSITION AND DIRECTION TO SPHERICAL POLAR COORDINATESREAC 80
C********* AT A DISTANCE S ALONG THE RAY                                REAC 81
      CALL CAR POL                                                      REAC 82
C********* AVOID THE RAY BEING SLIGHTLY UNDERGROUND                     REAC 83
      R(1)=AMAX1(R(1),EARTHR)                                           REAC 84
C********* ONE STEP INTEGRATION                                         REAC 85
      IF (N.LT.7) GO TO 31                                              REAC 86
      DO 30 NN=7,N                                                      REAC 87
   30 R(NN)=R(NN)+S*DROT(NN)                                            REAC 88
   31 T=T+S                                                             REAC 89
      CALL RINDEX                                                       REAC 90
C********* AT A PERIGEE                                                 REAC 91
      PERIGE=S.EQ.(-UP)                                                 REAC 92
C********* CORRECT MINOR ERRORS                                         REAC 93
      IF (PERIGE) R(4)=0.                                               REAC 94
C********* KEEP CONSISTENCY AFTER CORRECTING MINOR ERRORS               REAC 95
      DROT(1)=R(4)                                                      REAC 96
C********* ON THE GROUND                                                REAC 97
      GROUND=S.EQ.SG.AND.CROSSG                                         REAC 98
C********* AT THE RECEIVER HEIGHT                                       REAC 99
      THERE=S.EQ.SR.AND.CROSSR.AND..NOT.PERIGE                          REAC100
C********* AT A CLOSEST APPROACH TO THE RECEIVER HEIGHT                 REAC101
      MINDIS=PERIGE.AND.S.EQ.SR.AND.CROSSR                              REAC102
      RSPACE=SPACE                                                      REAC103
      V=SQRT(N2/(R(4)**2+R(5)**2+R(6)**2))                              REAC104
C********* RENORMALIZE THE WAVE NORMAL DIRECTION TO = SQRT(REAL(N**2))  REAC105
      R(4)=R(4)*V                                                       REAC106
      R(5)=R(5)*V                                                       REAC107
      R(6)=R(6)*V                                                       REAC108
      RSTART=1.                                                         REAC109
      IF (.NOT.SPACE) CALL PRINTR (8HENTR ION,8.)                       REAC110
      RETURN                                                            REAC111
         END                                                            REAC112-
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      SUBROUTINE POL CAR                                                POLC001
      DIMENSION XO(6),X(6),RO(4)                                        POLC002
      COMMON R(6) /COORD/ S                                             POLC003
      COMMON /CONST/ PI,PIT2,PID2,DUM(5)                                POLC004
C                                                                       POLC005
C          CONVERTS SPHERICAL COORDINATES TO CARTESIAN                  POLC006
      IF (R(5).EQ.0..AND.R(6).EQ.0.) GO TO 1                            POLC007
      VERT=0.                                                           POLC008
      SINA=SIN(R(2))                                                    POLC009
      COSA=SIN(PID2-R(2))                                               POLC010
      SINP=SIN(R(3))                                                    POLC011
      COSP=SIN(PID2-R(3))                                               POLC012
      XO(1)=R(1)*SINA*COSP                                              POLC013
      XO(2)=R(1)*SINA*SINP                                              POLC014
      XO(3)=R(1)*COSA                                                   POLC015
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      X(4)=R(4)*SINA*COSP+R(5)*COSA*COSP-R(6)*SINP                      POLC016
      X(5)=R(4)*SINA*SINP+R(5)*COSA*SINP+R(6)*COSP                      POLC017
      X(6)=R(4)*COSA-R(5)*SINA                                          POLC018
      RETURN                                                            POLC019
C           VERTICAL INCIDENCE                                          POLC020
    1 VERT=1.                                                           POLC021
      RO(1)=R(1)                                                        POLC022
      RO(2)=R(2)                                                        POLC023
      RO(3)=R(3)                                                        POLC024
      RO(4)=SIGN (1.,R(4))                                              POLC025
      RETURN                                                            POLC026
C                                                                       POLC027
C           STEPS THE RAY A DISTANCE S, AND THEN                        POLC028
C           CONVERTS CARTESIAN COORDINATES TO SPHERICAL COORDINATES     POLC029
      ENTRY CAR POL                                                     POLC030
      IF (VERT.NE.0.) GO TO 2                                           POLC031
      X(1)=XO(1)+S*X(4)                                                 POLC032
      X(2)=XO(2)+S*X(5)                                                 POLC033
      X(3)=XO(3)+S*X(6)                                                 POLC034
      TEMP=SQRT(X(1)**2+X(2)**2)                                        POLC035
      R(1)=SQRT(X(1)**2+X(2)**2+X(3)**2)                                POLC036
      R(2)=ATAN2(TEMP,X(3))                                             POLC037
      R(3)=ATAN2(X(2),X(1))                                             POLC038
      R(4)=(X(1)*X(4)+X(2)*X(5)+X(3)*X(6))/R(1)                         POLC039
      R(5)=(X(3)*(X(1)*X(4)+X(2)*X(5))-(X(1)**2+X(2)**2)*X(6))/         POLC040
     1 (R(1)*TEMP)                                                      POLC041
      R(6)=(X(1)*X(5)-X(2)*X(4))/TEMP                                   POLC042
      RETURN                                                            POLC043
C           VERTICAL INCIDENCE                                          POLC044
    2 R(1)=RO(1)+RO(4)*S                                                POLC045
      R(2)=RO(2)                                                        POLC046
      R(3)=RO(3)                                                        POLC047
      R(4)=RO(4)                                                        POLC048
      R(5)=0.                                                           POLC049
      R(6)=0.                                                           POLC050
      RETURN                                                            POLC051
         END                                                            POLC 52-
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      SUBROUTINE PRINTR(NWHY,CARD)                                      PRIN001
C           PRINTS OUTPUT AND PUNCHES RAYSETS WHEN REQUESTED            PRIN002
      DIMENSION G(3,3),G1(3,3),TYPE(3),HEADR1(20),HEADR2(20),UNITS(20), PRIN003
     1          HEAD1(20),HEAD2(20),UNIT(20),RPRINT(20),NPR(20)         PRIN004
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RAD,DUM(3)                       PRIN005
      COMMON /FLG/ NTYP,NEWWR,NEWWP,PENET,LINES,IHOP,HPUNCH             PRIN006
      COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,N2,N2I,PNP(10),POLAR(2),  PRIN007
     1             LPOLAR(2)                                            PRIN008
      COMMON R(20),T /WW/ ID(10),WO,W(400)                              PRIN009
      EQUIVALENCE (THETA,R(2)),(PHI,R(3))                               PRIN010
      EQUIVALENCE (EARTHR,W(2)),(XMTRH,W(3)),(TLAT,W(4)),(TLON,W(5)),   PRIN011
     1 (F,W(6)),(AZ1,W(10)),(BETA,W(14)),(RCVRH,W(20)),(HOP,W(22)),     PRIN012
     2 (PLAT,W(24)),(PLON,W(25)),(RAYSET,W(72))                         PRIN013
      LOGICAL SPACE,NEWWR,NEWWP,PENET                                   PRIN014
      REAL N2,N2I,LPOLAR                                                PRIN015
      COMPLEX PNP                                                       PRIN016
      DATA (TYPE=1HX,1HN,1HO)                                           PRIN017
     2,(HEADR1(7)=5H  PHAS),(HEADR2(7)=6HE PATH),(UNITS(7)=6H   KM ),   PRIN018
     3 (HEADR1(8)=5H  ABSO),(HEADR2(8)=6HRPTION),(UNITS(8)=6H   DB ),   PRIN019
     4 (HEADR1(9)=6H   DOP),(HEADR2(9)=6HPLER  ),(UNITS(9)=6H  C/S ),   PRIN020
     5 (HEADR1(10)=5H PATH ),(HEADR2(10)=6HLENGTH),(UNITS(10)=6H   KM ) PRIN023
      CALL RINDEX                                                       PRIN024
      IF (.NOT.NEWWP) GO TO 10                                          PRIN025
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C********* NEW W ARRAY -- REINITIALIZE                                      PRIN026
      NEWWP=.FALSE.                                                          PRIN027
      SPL=SIN (PLON-TLON)                                                    PRIN028
      CPL=SIN (PID2-(PLON-TLON))                                             PRIN029
      SP=SIN (PLAT)                                                          PRIN030
      CP=SIN (PID2-PLAT)                                                     PRIN031
      SL=SIN (TLAT)                                                          PRIN032
      CL=SIN (PID2-TLAT)                                                     PRIN033
C********* MATRIX TO ROTATE COORDINATES                                      PRIN034
      G(1,1)=CPL*SP*CL-CP*SL                                                 PRIN035
      G(1,2)=SPL*SP                                                          PRIN036
      G(1,3)=-SL*SP*CPL-CL*CP                                                PRIN037
      G(2,1)=-SPL*CL                                                         PRIN038
      G(2,2)=CPL                                                             PRIN039
      G(2,3)=SL*SPL                                                          PRIN040
      G(3,1)=CL*CP*CPL+SP*SL                                                 PRIN041
      G(3,2)=CP*SPL                                                          PRIN042
      G(3,3)=-SL*CP*CPL+SP*CL                                                PRIN043
      DENM=G(1,1)*G(2,2)*G(3,3)+G(1,2)*G(3,1)*G(2,3)+G(2,1)*G(3,2)*G(1,3PRIN044
     1)-G(2,2)*G(3,1)*G(1,3)-G(1,2)*G(2,1)*G(3,3)-G(1,1)*G(3,2)*G(2,3)  PRIN045
C********* THE MATRIX G1 IS THE INVERSE OF THE MATRIX G                      PRIN046
      G1(1,1)=(G(2,2)*G(3,3)-G(3,2)*G(2,3))/DENM                             PRIN047
      G1(1,2)=(G(3,2)*G(1,3)-G(1,2)*G(3,3))/DENM                             PRIN048
      G1(1,3)=(G(1,2)*G(2,3)-G(2,2)*G(1,3))/DENM                             PRIN049
      G1(2,1)=(G(3,1)*G(2,3)-G(2,1)*G(3,3))/DENM                             PRIN050
      G1(2,2)=(G(1,1)*G(3,3)-G(3,1)*G(1,3))/DENM                             PRIN051
      G1(2,3)=(G(2,1)*G(1,3)-G(1,1)*G(2,3))/DENM                             PRIN052
      G1(3,1)=(G(2,1)*G(3,2)-G(3,1)*G(2,2))/DENM                             PRIN053
      G1(3,2)=(G(3,1)*G(1,2)-G(1,1)*G(3,2))/DENM                             PRIN054
      G1(3,3)=(G(1,1)*G(2,2)-G(2,1)*G(1,2))/DENM                             PRIN055
      R0=EARTHR+XMTRH                                                        PRIN056
C********* CARTESIAN COORDINATES OF TRANSMITTER                              PRIN057
      XR=R0*G(1,1)                                                           PRIN058
      YR=R0*G(2,1)                                                           PRIN059
      ZR=R0*G(3,1)                                                           PRIN060
      CTHR=G(3,1)                                                            PRIN061
      STHR=SIN (ACOS (CTHR))                                                 PRIN062
      PHIR=ATAN2(YR,XR)                                                      PRIN063
      ALPH=ATAN2(G(3,2),G(3,3))                                              PRIN064
C*********                                                                   PRIN065
      NR=6                                                                   PRIN066
      NP=0                                                                   PRIN067
      DO 7 NN=7,20                                                           PRIN068
      IF (W(NN+50).EQ.0.) GO TO 7                                            PRIN069
C********* DEPENDENT VARIABLE NUMBER NN IS BEING INTEGRATED                  PRIN070
C********* NR IS THE NUMBER OF DEPENDENT VARIABLES BEING INTEGRATED          PRIN071
      NR=NR+1                                                                PRIN072
      IF (W(NN+50).NE.2.) GO TO 7                                            PRIN073
C********* DEPENDENT VARIABLE NUMBER NN IS BEING INTEGRATED AND PRINTED.PRIN074
C********* NP IS THE NUMBER OF DEPENDENT VARIABLES BEING INTEGRATED AND PRIN075
C********* PRINTED                                                           PRIN076
      NP=NP+1                                                                PRIN077
C********* SAVE THE INDEX OF THE DEPENDENT VARIABLE TO PRINT                 PRIN078
      NPR(NP)=NR                                                             PRIN079
      HEAD1(NP)=HEADR1(NN)                                                   PRIN080
      HEAD2(NP)=HEADR2(NN)                                                   PRIN081
      UNIT(NP)=UNITS(NN)                                                     PRIN082
    7 CONTINUE                                                               PRIN083
      NP1=MIN0(NP,3)                                                         PRIN084
      PDEV=ABSORB=DOPP=0.                                                    PRIN085
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C********* PRINT COLUMN HEADINGS AT THE BEGINNING OF EACH RAY          PRIN086
   10 IF (IHOP.NE.0) GO TO 12                                          PRIN087
      PRINT 1100, (HEAD1(NN),HEAD2(NN),NN=1,NP1)                       PRIN088
 1100 FORMAT (44X,7HAZIMUTH/43X,9HDEVIATION,8X,9HELEVATION/            PRIN089
     1 19X,16HHEIGHT     RANGE,1X,2(5X,12HXMTR   LOCAL),5X,26HPOLARIZATIPRIN090
     2ON    GROUP PATH,5A6,A5)                                         PRIN091
      PRINT 1150, (UNIT(NN),NN=1,NP1)                                  PRIN092
 1150 FORMAT (13X,2(8X,2HKM),2X,2(6X,3HDEG,5X,3HDEG),6X,12HREAL    IMAG,PRIN093
     1 7X,2HKM,4X,3(4X,A6,2X))                                         PRIN094
      IF (RAYSET.EQ.0.) GO TO 12                                       PRIN095
C********* PUNCH A TRANSMITTER RAYSET                                  PRIN096
      TLOND=TLON*DEGS                                                  PRIN097
      IF (TLOND.LT.0.) TLOND=TLOND+360.                                PRIN098
      TLATD=TLAT*DEGS                                                  PRIN099
      IF (TLATD.LT.0.) TLATD=TLATD+360.                                PRIN100
      AZ=AZ1*DEGS                                                      PRIN101
      EL=BETA*DEGS                                                     PRIN102
      NHOP=HOP                                                         PRIN103
      PUNCH 1200, ID(1),TYPE(NTYP),XMTRH,TLATD,TLOND,RCVRH,F,AZ,EL,POLARPRIN104
     1,NHOP,1HT                                                        PRIN105
 1200 FORMAT (A3,A1,4PF9.0,3P2F6.0,4P2F9.0,5P2F10.0,5X,2P2F5.0,I1,A1)  PRIN106
C*********                                                            PRIN107
   12 V=0.                                                             PRIN108
      IF (N2.NE.0.) V=(R(4)**2+R(5)**2+R(6)**2)/N2-1.                  PRIN109
      H=R(1)-EARTHR                                                    PRIN110
      STH=SIN (THETA)                                                  PRIN111
      CTH=SIN (PID2-THETA)                                             PRIN112
C********* CARTESIAN COORDINATES OF RAY POINT, ORIGIN AT TRANSMITTER   PRIN113
      XP=R(1)*STH*SIN (PID2-PHI)-XR                                    PRIN114
      YP=R(1)*STH*SIN (PHI)-YR                                         PRIN115
      ZP=R(1)*CTH-ZR                                                   PRIN116
C********* CARTESIAN COORDINATES OF RAY POINT, ORIGIN AT TRANSMITTER ANDPRIN117
C********* ROTATED                                                     PRIN118
      EPS=XP*G1(1,1)+YP*G1(1,2)+ZP*G1(1,3)                             PRIN119
      ETA=XP*G1(2,1)+YP*G1(2,2)+ZP*G1(2,3)                             PRIN120
      ZETA=XP*G1(3,1)+YP*G1(3,2)+ZP*G1(3,3)                            PRIN121
      RCE2=ETA**2+ZETA**2                                              PRIN122
      RCE=SQRT (RCE2)                                                  PRIN123
C********* GROUND RANGE                                                PRIN124
      RANGE=EARTHR*ATAN2(RCE,EARTHR+EPS+XMTRH)                         PRIN125
C********* ANGLE OF WAVE NORMAL WITH LOCAL HORIZONTAL                  PRIN126
      ELL=ATAN2(R(4),SQRT (R(5)**2+R(6)**2))*DEGS                      PRIN127
C********* STRAIGHT LINE DISTANCE FROM TRANSMITTER TO RAY POINT        PRIN128
      SR=SQRT (RCE2+EPS**2)                                            PRIN129
      IF (NP.LT.1) GO TO 16                                            PRIN130
      DO 15 I=1,NP                                                     PRIN131
      NN=NPR(I)                                                        PRIN132
   15 RPRINT(I)=R(NN)                                                  PRIN133
   16 IF (SR.GE.1.E-6) GO TO 20                                        PRIN134
C********* TOO CLOSE TO TRANSMITTER TO CALCULATE DIRECTION FROM        PRIN135
C********* TRANSMITTER                                                 PRIN136
      PRINT 1500, V,NWHY,H,RANGE,ELL,POLAR,T,(RPRINT(NN),NN=1,NP1)     PRIN137
 1500 FORMAT (1X,E6.0,1X,A8,F10.4,F11.4,26X,F8.3,F9.3,F8.3,4F12.4)     PRIN138
      GO TO 40                                                         PRIN139
C********* ELEVATION ANGLE OF RAY POINT FROM TRANSMITTER               PRIN140
   20 EL=ATAN2(EPS,RCE)*DEGS                                           PRIN141
      IF (RCE.GE.1.E-6) GO TO 30                                       PRIN142
C********* NEARLY DIRECTLY ABOVE OR BELOW TRANSMITTER. CAN NOT CALCULATEPRIN143
C********* AZIMUTH DIRECTION FROM TRANSMITTER ACCURATELY               PRIN144
      PRINT 2500, V,NWHY,H,RANGE,EL,ELL,POLAR,T,(RPRINT(NN),NN=1,NP1)  PRIN145
 2500 FORMAT (1X,E6.0,1X,A8,F10.4,F11.4,17X,F9.3,F8.3,F9.3,F8.3,       PRIN146
     1 4F12.4)                                                         PRIN147
      GO TO 40                                                         PRIN148
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C********* AZIMUTH ANGLE OF RAY POINT FROM TRANSMITTER                    PRIN149
   30 ANGA=ATAN2(ETA,ZETA)                                                PRIN150
      AZDEV=180.-AMOD(540.-(AZ1-ANGA)*DEGS,360.)                          PRIN151
      IF (R(5).NE.0..OR.R(6).NE.0.) GO TO 34                              PRIN152
C********* WAVE NORMAL IS VERTICAL, SO AZIMUTH DIRECTION CANNOT BE        PRIN153
C********* CALCULATED                                                     PRIN154
      PRINT 3000, V,NWHY,H,RANGE,AZDEV,EL,ELL,POLAR,T,(RPRINT(NN),NN=1,   PRIN155
     1 NP1)                                                               PRIN156
 3000 FORMAT (1X,E6.0,1X,A8,F10.4,F11.4,F9.3,8X,F9.3,F8.3,F9.3,F8.3,     PRIN157
     1 4F12.4)                                                            PRIN158
      GO TO 40                                                            PRIN159
   34 ANA=ANGA-ALPH                                                       PRIN160
      SANA=SIN (ANA)                                                      PRIN161
      SPHI=SANA*STHR/STH                                                  PRIN162
      CPHI=-SIN (PID2-ANA)*SIN (PID2-(PHI-PHIR))+SANA*SIN (PHI-PHIR)      PRIN163
     1 *CTHR                                                              PRIN164
      AZA=180.-AMOD (540.-(ATAN2(SPHI,CPHI)-ATAN2(R(6),R(5)))*DEGS,360.)PRIN165
      PRINT 3500, V,NWHY,H,RANGE,AZDEV,AZA,EL,ELL,POLAR,T,(RPRINT(NN),NNPRIN166
     1 =1,NP1)                                                            PRIN167
 3500 FORMAT (1X,E6.0,1X,A8,F10.4,F11.4,2(F9.3,F8.3),F9.3,F8.3,          PRIN168
     1 4F12.4)                                                            PRIN169
C*********                                                               PRIN170
   40 LINES=LINES+1                                                       PRIN171
      IF (NP.LE.3) GO TO 45                                               PRIN172
C********* ADDITIONAL LINE TO PRINT REMAINING DEPENDENT INTEGRATION       PRIN173
C********* VARIABLES                                                      PRIN174
      PRINT 4000, (RPRINT(NN),NN=4,NP)                                    PRIN175
 4000 FORMAT (99X,3F12.4)                                                 PRIN176
      LINES=LINES+1                                                       PRIN177
   45 IF (CARD.EQ.0.) RETURN                                              PRIN178
C                                                                         PRIN179
C********* PUNCH A RAYSET                                                 PRIN180
      IF (AZDEV.LT.-90.) AZDEV=AZDEV+360.                                 PRIN181
      IF (AZA.LT.-90.) AZA=AZA+360.                                       PRIN182
      TDEV=T-SR                                                           PRIN183
      NR=6                                                                PRIN184
      IF (W(57).EQ.0.) GO TO 47                                           PRIN185
C********* PHASE PATH                                                     PRIN186
      NR=NR+1                                                             PRIN187
      PDEV=R(NR)-SR                                                       PRIN188
   47 IF (W(58).EQ.0.) GO TO 48                                           PRIN189
C********* ABSORPTION                                                     PRIN190
      NR=NR+1                                                             PRIN191
      ABSORB=R(NR)                                                        PRIN192
C********* DOPPLER SHIFT                                                  PRIN193
   48 IF (W(59).NE.0.) DOPP=R(NR+1)                                       PRIN194
      PUNCH 4500, HPUNCH,RANGE,AZDEV,AZA,ELL,SR,TDEV,PDEV,ABSORB,DOPP,    PRIN195
     1 POLAR,IHOP,NWHY                                                    PRIN196
 4500 FORMAT (4P2F9.0,3P3F6.0,3PF8.0,3P4F6.0,2P2F5.0,I1,A1)               PRIN197
      RETURN                                                              PRIN198
         END                                                              PRIN199-
```

INPUT PARAMETER FORM FOR PLOTTING THE PROJECTION OF THE RAY PATH ON A VERTICAL PLANE

Coordinates of the left edge of the graph:

Latitude = _____ rad / deg / km north (W83)

Longitude = _____ rad / deg / km east (W84)

Coordinates of the right edge of the graph:

Latitude = _____ rad / deg / km north (W85)

Longitude = _____ rad / deg / km east (W86)

Height above the ground of the bottom of the graph = _____ km (W88)

Distance between tic marks = _____ rad / deg / km (W87)

(W81 = 1.)

INPUT PARAMETER FORM FOR PLOTTING THE PROJECTION OF THE RAY PATH ON THE GROUND

Coordinates of the left edge of the graph:

Latitude = _____ rad / deg / km north (W83)

Longitude = _____ rad / deg / km east (W84)

Coordinates of the right edge of the graph:

Latitude = _____ rad / deg / km north (W85)

Longitude = _____ rad / deg / km east (W86)

Factor to expand lateral deviation scale by = _____ (W82)

Distance between tic marks on range scale = _____ rad / deg / km (W87)

(W81 = 2.)

```
      SUBROUTINE RAYPLT                                                 YPLT001
C        REPLACES SUBROUTINES RAYPLT,PLOT, AND LABPLT IF PLOTS ARE      YPLT002
C        NOT WANTED OR IF A PLOTTER IS NOT AVAILABLE                    YPLT003
      COMMON /WW/ ID(10),W0,W(400)                                      YPLT004
      EQUIVALENCE (PLT,W(81))                                           YPLT005
      PLT=0.                                                            YPLT006
      ENTRY ENDPLT                                                      YPLT007
      RETURN                                                            YPLT008
         END                                                            YPLT  9-
```

```
      SUBROUTINE RAYPLT                                                 RAYP001
C        W(81)=1. PLOTS PROJECTION OF RAYPATH ON VERTICAL PLANE         RAYP002
C              =2. PLOTS PROJECTION OF RAYPATH ON GROUND                RAYP003
      COMMON /PLT/ XL,XR,YB,YT,RESET                                    RAYP004
      COMMON /CONST/ PI,PIT2,PID2,DUM(5)                                RAYP005
      COMMON /FLG/ NTYP,NEWWR,NEWWP,PENET,LINES,IHOP,HPUNCH             RAYP006
      COMMON R(6) /WW/ ID(10),W0,W(400)                                 RAYP007
      EQUIVALENCE (TH,R(2)),(PH,R(3))                                   RAYP008
      EQUIVALENCE (EARTHR,W(2)),(PLAT,W(24)),(PLON,W(25)),(PLT,W(81)),  RAYP009
     1 (FACTR,W(82)),(LLAT,W(83)),(LLON,W(84)),(RLAT,W(85)),(RLON,W(86))RAYP010
     2,(TIC,W(87)),(HB,W(88))                                           RAYP011
      REAL LLAT,LLON,LTIC                                               RAYP012
      LOGICAL NEWWR,NEWWP,PENET                                         RAYP013
      IF (.NOT.NEWWR) GO TO 5                                           RAYP014
C                                                                       RAYP015
C           NEW W ARRAY -- REINITIALIZE                                 RAYP016
      NEWWR=.FALSE.                                                     RAYP017
      RESET=1.                                                          RAYP018
CONVERT COORDINATES OF VERTICAL PLANE FROM GEOGRAPHIC TO GEOMAGNETIC    RAYP019
      SW=SIN (PLAT)                                                     RAYP020
      CW=SIN (PID2-PLAT)                                                RAYP021
      SLM=SIN (LLAT)                                                    RAYP022
      CLM=SIN (PID2-LLAT)                                               RAYP023
      SRM=SIN (RLAT)                                                    RAYP024
      CRM=SIN (PID2-RLAT)                                               RAYP025
      CDPHI=SIN (PID2-(LLON-PLON))                                      RAYP026
      PHL=ATAN2(SIN (LLON-PLON)*CLM,CDPHI*SW*CLM-CW*SLM)                RAYP027
      CTHL=CDPHI*CW*CLM+SW*SLM                                          RAYP028
      STHL=SIN (ACOS (CTHL))                                            RAYP029
      CDPHI=SIN (PID2-(RLON-PLON))                                      RAYP030
      PHR=ATAN2(SIN (RLON-PLON)*CRM,CDPHI*SW*CRM-CW*SRM)                RAYP031
      CTHR=CDPHI*CW*CRM+SW*SRM                                          RAYP032
      STHR=SIN (ACOS (CTHR))                                            RAYP033
      CLR=CTHL*CTHR+STHL*STHR*SIN (PID2-(PHL-PHR))                      RAYP034
      SLR=SQRT (1.-CLR**2)                                              RAYP035
      IF (PLT.EQ.2.) GO TO 3                                            RAYP036
      FACTR=1.                                                          RAYP037
      R0=EARTHR+HB                                                      RAYP038
      ALPHA=.5*ACOS (CLR)                                               RAYP039
      XR=R0*SIN (ALPHA)                                                 RAYP040
      XL=-XR                                                            RAYP041
      YB=R0*SIN (PID2-ALPHA)                                            RAYP042
      YT=YB+2.*XR                                                       RAYP043
      GO TO 5                                                           RAYP044
    3 IF (FACTR.EQ.0.) FACTR=1.                                         RAYP045
      ALPH1=ATAN2(STHR*SIN (PHR-PHL),(CTHR-CTHL*CLR)/STHL)              RAYP046
      XL=0.                                                             RAYP047
      XR=EARTHR*ACOS (CLR)                                              RAYP048
      YT=0.5*XR/FACTR                                                   RAYP049
      YB=-YT                                                            RAYP050
```

```
C                                                                       RAYP051
    5 STH=SIN (TH)                                                      RAYP052
      CTH=SIN (PID2-TH)                                                 RAYP053
      CR=CTHR*CTH+STHR*STH*SIN (PID2-(PHR-PH))                          RAYP054
      CL=CTHL*CTH+STHL*STH*SIN (PID2-(PHL-PH))                          RAYP055
      CEA=ATAN2(CR-CL*CLR,CL*SLR)                                       RAYP056
      NEW=1                                                             RAYP057
      IF (IHOP.NE.0) NEW=0                                              RAYP058
      IF (PLT.EQ.2.) GO TO 10                                           RAYP059
      CALL PLOT (R(1)*SIN(CEA-ALPHA),R(1)*SIN(PID2-(CEA-ALPHA)),NEW)    RAYP060
      RETURN                                                            RAYP061
   10 SL=SQRT (1.-CL**2)                                                RAYP062
      TMP1=STH*SIN (PH-PHL)                                             RAYP063
      TMP2=(CTH-CTHL*CL)/STHL                                           RAYP064
      ALPH2=0.                                                          RAYP065
      IF (TMP1.NE.0..OR.TMP2.NE.0.) ALPH2=ATAN2(TMP1,TMP2)              RAYP066
      CALL PLOT (EARTHR*CEA,EARTHR*ASIN(SL*SIN (ALPH1-ALPH2)),NEW)      RAYP067
      RETURN                                                            RAYP068
C                                                                       RAYP069
C           DRAW AXES AND CALL FOR LABELING AND TERMINATION OF THIS PLOT RAYP070
      ENTRY ENDPLT                                                      RAYP071
      TICKX=0.01*(YT-YB)                                                RAYP072
      IF (PLT.EQ.2.) GO TO 25                                           RAYP073
      R1=EARTHR-TICKX                                                   RAYP074
      X=XL                                                              RAYP075
      Y=YB                                                              RAYP076
      CALL PLOT (X,Y,1)                                                 RAYP077
      NTIC=2                                                            RAYP078
      IF (TIC.NE.0.) NTIC=NTIC+2.*ALPHA/TIC                             RAYP079
      NLINE=MAX0 (1,100/NTIC)                                           RAYP080
      DO 20 I=1,NTIC                                                    RAYP081
      ANG=-ALPHA+(I-1)*TIC                                              RAYP082
      CALL PLOT (R1*SIN (ANG),R1*SIN (PID2-ANG),0)                      RAYP083
      CALL PLOT (X,Y,0)                                                 RAYP084
      DO 20 J=1,NLINE                                                   RAYP085
      ANG=ANG+TIC/NLINE                                                 RAYP086
      X=EARTHR*SIN (ANG)                                                RAYP087
      Y=EARTHR*SIN (PID2-ANG)                                           RAYP088
   20 CALL PLOT (X,Y,0)                                                 RAYP089
      CALL PLOT (XR,YB,0)                                               RAYP090
      GO TO 50                                                          RAYP091
   25 DTIC=TIC*EARTHR                                                   RAYP092
      LTIC=DTIC/FACTR                                                   RAYP093
      TICY=XL+0.01*(XR-XL)                                              RAYP094
      NTIC=YT/LTIC                                                      RAYP095
      TIC1=-LTIC*NTIC                                                   RAYP096
      CALL PLOT (XL,YB,1)                                               RAYP097
      NTIC=2*NTIC+1                                                     RAYP098
      DO 30 I=1,NTIC                                                    RAYP099
      Y=TIC1+(I-1)*LTIC                                                 RAYP100
      CALL PLOT (XL,Y,0)                                                RAYP101
      CALL PLOT (TICY,Y,0)                                              RAYP102
   30 CALL PLOT (XL,Y,0)                                                RAYP103
      CALL PLOT (XL,YT,0)                                               RAYP104
      CALL PLOT (XL,0.,1)                                               RAYP105
      NTIC=(XR-XL)/DTIC                                                 RAYP106
      DO 40 I=1,NTIC                                                    RAYP107
      X=I*DTIC                                                          RAYP108
      CALL PLOT (X,0.,0)                                                RAYP109
      CALL PLOT (X,TICKX,0)                                             RAYP110
   40 CALL PLOT (X,0.,0)                                                RAYP111
      CALL PLOT (XR,0.,0)                                               RAYP112
   50 CALL LABPLT                                                       RAYP113
      CALL PLTEND                                                       RAYP114
      RETURN                                                            RAYP115
         END                                                            RAYP116-
```

```
      SUBROUTINE PLOT (X,Y,NEW)                                         PLOT001
      COMMON /PLT/ XMIN0,XMAX0,YMIN0,YMAX0,RESET                        PLOT002
      COMMON /DD/ INT,IOR,IT,IS,IC,ICC,IX,IY                            PLOT003
      DATA (INITAL=1),(MINX=0),(MINY=0),(MAXX=1023),(MAXY=1023),        PLOT004
     1 (MINX0=23),(MINY0=23),(MAXX0=1023),(MAXY0=1023)                  PLOT005
C                                                                       PLOT006
C           INITIALIZE LIBRARY PLOTTING ROUTINES                        PLOT007
      IF (INITAL.EQ.0) GO TO 1                                          PLOT008
      INITAL=0                                                          PLOT009
      CALL DDINIT (1,1H )                                               PLOT010
C                                                                       PLOT011
C           COMPUTE SCALE FACTORS                                       PLOT012
    1 IF (RESET.EQ.0.) GO TO 5                                          PLOT013
      RESET=0.                                                          PLOT014
      XSCALE=(MAXX0-MINX0)/(XMAX0-XMIN0)                                PLOT015
      YSCALE=(MAXY0-MINY0)/(YMAX0-YMIN0)                                PLOT016
      XMIN=XMIN0-(MINX0-MINX)/XSCALE                                    PLOT017
      YMIN=YMIN0-(MINY0-MINY)/YSCALE                                    PLOT018
      XMAX=XMAX0+(MAXX-MAXX0)/XSCALE                                    PLOT019
      YMAX=YMAX0+(MAXY-MAXY0)/YSCALE                                    PLOT020
C                                                                       PLOT021
C           START A NEW LINE                                            PLOT022
    5 IF (NEW.EQ.0) GO TO 10                                            PLOT023
      IX=MINX0+(X-XMIN0)*XSCALE                                         PLOT024
      IY=MINY0+(Y-YMIN0)*YSCALE                                         PLOT025
      IF (IX.GE.MINX.AND.IX.LE.MAXX.AND.IY.GE.MINY.AND.IY.LE.MAXY)      PLOT026
     1 CALL DDBP                                                        PLOT027
      GO TO 50                                                          PLOT028
C                                                                       PLOT029
C           HORIZONTAL DISPLACEMENT                                     PLOT030
   10 XS=X-XOLD                                                         PLOT031
      YS=Y-YOLD                                                         PLOT032
      IF (XS) 11,12,16                                                  PLOT033
C           NEGATIVE                                                    PLOT034
   11 X1=XMAX                                                           PLOT035
      X2=XMIN                                                           PLOT036
      GO TO 20                                                          PLOT037
C           ZERO                                                        PLOT038
   12 IF (YS) 13,50,14                                                  PLOT039
   13 S1=(YMAX-YOLD)/YS                                                 PLOT040
      S2=(YMIN-YOLD)/YS                                                 PLOT041
      GO TO 40                                                          PLOT042
   14 S1=(YMIN-YOLD)/YS                                                 PLOT043
      S2=(YMAX-YOLD)/YS                                                 PLOT044
      GO TO 40                                                          PLOT045
C           POSITIVE                                                    PLOT046
   16 X1=XMIN                                                           PLOT047
      X2=XMAX                                                           PLOT048
C                                                                       PLOT049
C           VERTICAL DISPLACEMENT                                       PLOT050
   20 IF (YS) 21,22,26                                                  PLOT051
C           NEGATIVE                                                    PLOT052
   21 Y1=YMAX                                                           PLOT053
      Y2=YMIN                                                           PLOT054
      GO TO 30                                                          PLOT055
C           ZERO                                                        PLOT056
   22 S1=(X1-XOLD)/XS                                                   PLOT057
      S2=(X2-XOLD)/XS                                                   PLOT058
      GO TO 40                                                          PLOT059
C           POSITIVE                                                    PLOT060
   26 Y1=YMIN                                                           PLOT061
      Y2=YMAX                                                           PLOT062
C                                                                       PLOT063
   30 S1=AMAX1((X1-XOLD)/XS,(Y1-YOLD)/YS)                               PLOT064
      S2=AMIN1((X2-XOLD)/XS,(Y2-YOLD)/YS)                               PLOT065
```

```
C                                                                       PLOT066
C          PLOT LINE -- CHECKING FOR BORDER CROSSINGS                   PLOT067
   40 S=SQRT(XS**2+YS**2)                                               PLOT068
      IF (S2.LT.0..OR.S*S1-S.GT.0.) GO TO 50                            PLOT069
      IF (S1.LT.0.) GO TO 42                                            PLOT070
C          PREVIOUS POINT OFF GRAPH                                     PLOT071
      IX=MINX0+(XOLD+XS*S1-XMIN0)*XSCALE+0.5                            PLOT072
      IY=MINY0+(YOLD+YS*S1-YMIN0)*YSCALE+0.5                            PLOT073
      CALL DDBP                                                         PLOT074
   42 IF (S*S2-S.GT.0.) GO TO 44                                        PLOT075
C          CURRENT POINT OFF GRAPH                                      PLOT076
      IX=MINX0+(XOLD+XS*S2-XMIN0)*XSCALE+0.5                            PLOT077
      IY=MINY0+(YOLD+YS*S2-YMIN0)*YSCALE+0.5                            PLOT078
      CALL DDVC                                                         PLOT079
      GO TO 50                                                          PLOT080
C          CURRENT POINT ON GRAPH                                       PLOT081
   44 IX=MINX0+(X-XMIN0)*XSCALE+0.5                                     PLOT082
      IY=MINY0+(Y-YMIN0)*YSCALE+0.5                                     PLOT083
      CALL DDVC                                                         PLOT084
C                                                                       PLOT085
C          EXIT ROUTINE                                                 PLOT086
   50 XOLD=X                                                            PLOT087
      YOLD=Y                                                            PLOT088
      RETURN                                                            PLOT089
C                                                                       PLOT090
C          TERMINATE THE CURRENT PLOT                                   PLOT091
      ENTRY PLTEND                                                      PLOT092
      CALL DDFR                                                         PLOT093
      RETURN                                                            PLOT094
         END                                                            PLOT 95-
```

```
      SUBROUTINE LABPLT                                                 LABP001
C          LABEL THE CURRENT PLOT                                       LABP002
      DIMENSION LABEL(9),TYPE(3)                                        LABP003
      COMMON /DD/ INT,IOR,IT,IS,IC,ICC,IX,IY                            LABP004
      COMMON /CONST/ PI,PIT2,PID2,DEGS,DUM(4)                           LABP005
      COMMON /FLG/ NTYP,NEWWR,NEWWP,PENET,LINES,IHOP,HPUNCH             LABP006
      COMMON /WW/ ID(10),W0,W(400)                                      LABP007
      EQUIVALENCE (EARTHR,W(2)),(F,W(6)),(AZ1,W(10)),(PLT,W(81)),       LABP008
     1 (FACTR,W(82)),(TIC,W(87))                                        LABP009
      LOGICAL NEWWR,NEWWP,PENET                                         LABP010
      REAL LTIC                                                         LABP011
      DATA (TYPE=8HEXTRAORD,8HNO FIELD,8HORDINARY)                      LABP012
      IOR=IT=0                                                          LABP013
      IS=2                                                              LABP014
      IX=0      $  IY=1023  $  CALL DDTAB  $  CALL DDTEXT (7,ID)        LABP015
      NDATE=IDATE(0)                                                    LABP016
      CALL DDTEXT (1,NDATE)                                             LABP017
      AZA=AZ1*DEGS                                                      LABP018
      DTIC=TIC*EARTHR                                                   LABP019
      ENCODE (72,1000,LABEL)  F,AZA,TYPE(NTYP),DTIC                     LABP020
 1000 FORMAT (3HF =,F7.3,6H, AZ =,F7.2,2H, ,A8,2H, ,F7.2,24H KM BETWEEN LABP021
     1TICK MARKS*.)                                                     LABP022
      IX=0      $  IY=991   $  CALL DDTAB  $  CALL DDTEXT (9,LABEL)     LABP023
      IF (PLT.EQ.1.) RETURN                                             LABP024
      LTIC=DTIC/FACTR                                                   LABP025
      ENCODE (32,2000,LABEL)  LTIC                                      LABP026
 2000 FORMAT (F7.2,24H KM BETWEEN TICK MARKS*.)                         LABP027
      IOR=1                                                             LABP028
      IX=0      $  IY=0     $  CALL DDTAB  $  CALL DDTEXT (4,LABEL)     LABP029
      IOR=0                                                             LABP030
      RETURN                                                            LABP031
         END                                                            LABP 32-
```

```
      SUBROUTINE RKAM                                                   RKAM001
C        NUMERICAL INTEGRATION OF DIFFERENTIAL EQUATIONS                RKAM002
      COMMON /RK/ NN,SPACE,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART     RKAM003
      COMMON Y(20),T,STEP,DYDT(20)                                      RKAM004
      DIMENSION DELY(4,20),BET(4),XV(5),FV(4,20),YU(5,20)               RKAM005
      TYPE DOUBLE YU                                                    RKAM006
      IF (RSTART.EQ.0.) GO TO 1000                                      RKAM007
      LL=MM=1                                                           RKAM008
      IF (MODE.EQ.1) MM=4                                               RKAM009
      ALPHA=T                                                           RKAM010
      EPM=0.0                                                           RKAM011
      BET(1)=BET(2)=0.5                                                 RKAM012
      BET(3)=1.0                                                        RKAM013
      BET(4)=0.0                                                        RKAM014
      STEP=SPACE                                                        RKAM015
      R=19.0/270.0                                                      RKAM016
      XV(MM)=T                                                          RKAM017
      IF (E1MIN.LE.0.) E1MIN=E1MAX/55.                                  RKAM018
      IF (FACT.LE.0.) FACT=0.5                                          RKAM019
      CALL HAMLTN                                                       RKAM020
      DO 320 I=1,NN                                                     RKAM021
      FV(MM,I)=DYDT(I)                                                  RKAM022
  320 YU(MM,I)=Y(I)                                                     RKAM023
      RSTART=0.                                                         RKAM024
      GO TO 1001                                                        RKAM025
 1000 IF (MODE.NE.1) GO TO 2000                                         RKAM026
C                                                                       RKAM027
C          RUNGE-KUTTA                                                  RKAM028
 1001 DO 1034 K=1,4                                                     RKAM029
      DO 1350 I=1,NN                                                    RKAM030
      DELY(K,I)=STEP*FV(MM,I)                                           RKAM031
      Z=YU(MM,I)                                                        RKAM032
 1350 Y(I)=Z+BET(K)*DELY(K,I)                                           RKAM033
      T=BET(K)*STEP+XV(MM)                                              RKAM034
      CALL HAMLTN                                                       RKAM035
      DO 1034 I=1,NN                                                    RKAM036
 1034 FV(MM,I)=DYDT(I)                                                  RKAM037
      DO 1039 I=1,NN                                                    RKAM038
      DEL=(DELY(1,I)+2.0*DELY(2,I)+2.0*DELY(3,I)+DELY(4,I))/6.0         RKAM039
 1039 YU(MM+1,I)=YU(MM,I)+DEL                                           RKAM040
      MM=MM+1                                                           RKAM041
      XV(MM)=XV(MM-1)+STEP                                              RKAM042
      DO 1400 I=1,NN                                                    RKAM043
 1400 Y(I)=YU(MM,I)                                                     RKAM044
      T=XV(MM)                                                          RKAM045
      CALL HAMLTN                                                       RKAM046
      IF (MODE.EQ.1) GO TO 42                                           RKAM047
      DO 150 I=1,NN                                                     RKAM048
  150 FV(MM,I)=DYDT(I)                                                  RKAM049
      IF (MM.LE.3) GO TO 1001                                           RKAM050
C                                                                       RKAM051
C          ADAMS-MOULTON                                                RKAM052
 2000 DO 2048 I=1,NN                                                    RKAM053
      DEL=STEP*(55.*FV(4,I)-59.*FV(3,I)+37.*FV(2,I)-9.*FV(1,I))/24.     RKAM054
      Y(I)=YU(4,I)+DEL                                                  RKAM055
 2048 DELY(1,I)=Y(I)                                                    RKAM056
      T=XV(4)+STEP                                                      RKAM057
      CALL HAMLTN                                                       RKAM058
      XV(5)=T                                                           RKAM059
      DO 2051 I=1,NN                                                    RKAM060
      DEL=STEP*(9.*DYDT(I)+19.*FV(4,I)-5.*FV(3,I)+FV(2,I))/24.          RKAM061
      YU(5,I)=YU(4,I)+DEL                                               RKAM062
 2051 Y(I)=YU(5,I)                                                      RKAM063
      CALL HAMLTN                                                       RKAM064
      IF (MODE.LE.2) GO TO 42                                           RKAM065
```

```
C                                                                       RKAM066
C           ERROR ANALYSIS                                              RKAM067
      SSE=0.0                                                           RKAM068
      DO 3033 I=1,NN                                                    RKAM069
      EPSIL=R*ABS(Y(I)-DELY(1,I))                                       RKAM070
      IF (MODE.EQ.3.AND.Y(I).NE.0.) EPSIL=EPSIL/ABS(Y(I))               RKAM071
      IF (SSE.LT.EPSIL) SSE=EPSIL                                       RKAM072
 3033 CONTINUE                                                          RKAM073
      IF (E1MAX.GT.SSE) GO TO 3035                                      RKAM074
      IF (ABS(STEP).LE.E2MIN) GO TO 42                                  RKAM075
      LL=MM=1                                                           RKAM076
      STEP=STEP*FACT                                                    RKAM077
      GO TO 1001                                                        RKAM078
 3035 IF (LL.LE.1.OR.SSE.GE.E1MIN.OR.E2MAX.LE.ABS(STEP)) GO TO 42       RKAM079
      LL=2                                                              RKAM080
      MM=3                                                              RKAM081
      XV(2)=XV(3)                                                       RKAM082
      XV(3)=XV(5)                                                       RKAM083
      DO 5363 I=1,NN                                                    RKAM084
      FV(2,I)=FV(3,I)                                                   RKAM085
      FV(3,I)=DYDT(I)                                                   RKAM086
      YU(2,I)=YU(3,I)                                                   RKAM087
 5363 YU(3,I)=YU(5,I)                                                   RKAM088
      STEP=2.0*STEP                                                     RKAM089
      GO TO 1001                                                        RKAM090
C                                                                       RKAM091
C           EXIT ROUTINE                                                RKAM092
   42 LL=2                                                              RKAM093
      MM=4                                                              RKAM094
      DO 12 K=1,3                                                       RKAM095
      XV(K)=XV(K+1)                                                     RKAM096
      DO 12 I=1,NN                                                      RKAM097
      FV(K,I)=FV(K+1,I)                                                 RKAM098
   12 YU(K,I)=YU(K+1,I)                                                 RKAM099
      XV(4)=XV(5)                                                       RKAM100
      DO 52 I=1,NN                                                      RKAM101
      FV(4,I)=DYDT(I)                                                   RKAM102
   52 YU(4,I)=YU(5,I)                                                   RKAM103
      IF (MODE.LE.2) RETURN                                             RKAM104
      E=ABS(XV(4)-ALPHA)                                                RKAM105
      IF (E.LE.EPM) GO TO 2000                                          RKAM106
      EPM=E                                                             RKAM107
      RETURN                                                            RKAM108
         END                                                            RKAM109-
```

```
      SUBROUTINE HAMLTN                                                 HAML001
C********* CALCULATES HAMILTONS EQUATIONS FOR RAY TRACING              HAML002
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RAD,K,C,LOGTEN                   HAML003
      COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,KAY2,KAY2I,               HAML004
     1              H,HI,PHPT,PHPTI,PHPR,PHPRI,PHPTH,PHPTHI,PHPPH,PHPPHIHAML005
     2, PHPOM,PHPOMI,PHPKR,PHPKRI,PHPKTH,PHPKTI, PHPKPH,PHPKPI          HAML006
     3 ,KPHPK,KPHPKI,POLAR,POLARI,LPOLAR,LPOLRI                         HAML007
      COMMON R(20),T,STP,DRDT(20) /WW/ ID(10),W0,W(400)                 HAML008
      EQUIVALENCE (TH,R(2)),(PH,R(3)),(KR,R(4)),(KTH,R(5)),(KPH,R(6)),  HAML009
     1 (DTHDT,DRDT(2)),(DPHDT,DRDT(3)),(DKRDT,DRDT(4)),(DKTHDT,DRDT(5)),HAML010
     2 (DKPHDT,DRDT(6)),(F,W(6))                                        HAML011
      REAL KR,KTH,KPH,KPHPK,KPHPKI,LPOLAR,LPOLRI,LOGTEN,K,KAY2,KAY2I    HAML012
      OM=PIT2*1.E6*F                                                    HAML013
      STH=SIN(TH)                                                       HAML014
      CTH=SIN(PID2-TH)                                                  HAML015
      RSTH=R(1)*STH                                                     HAML016
      RCTH=R(1)*CTH                                                     HAML017
      CALL RINDEX                                                       HAML018
      DRDT=-PHPKR/(PHPOM*C)                                             HAML019
      DTHDT=-PHPKTH/(PHPOM*R(1)*C)                                      HAML020
      DPHDT=-PHPKPH/(PHPOM*RSTH*C)                                      HAML021
      DKRDT=PHPR/(PHPOM*C)+KTH*DTHDT+KPH*STH*DPHDT                      HAML022
      DKTHDT=(PHPTH/(PHPOM*C)-KTH*DRDT+KPH*RCTH*DPHDT)/R(1)             HAML023
      DKPHDT=(PHPPH/(PHPOM*C)-KPH*STH*DRDT-KPH*RCTH*DTHDT)/RSTH         HAML024
      NR=6                                                              HAML025
C********* PHASE PATH                                                  HAML026
      IF (W(57).EQ.0.) GO TO 10                                         HAML027
      NR=NR+1                                                           HAML028
      DRDT(NR)=-     KPHPK/PHPOM/OM                                     HAML029
C********* ABSORPTION                                                  HAML030
   10 IF (W(58).EQ.0.) GO TO 15                                         HAML031
      NR=NR+1                                                           HAML032
      DRDT(NR)= 10./LOGTEN*KPHPK*KAY2I/(KR*KR+KTH*KTH+KPH*KPH)/PHPOM/C HAML033
C********* DOPPLER SHIFT                                               HAML034
   15 IF (W(59).EQ.0.) GO TO 20                                         HAML035
      NR=NR+1                                                           HAML036
      DRDT(NR)=-PHPT/PHPOM/C/PIT2                                       HAML037
C********* GEOMETRICAL PATH LENGTH                                     HAML038
   20 IF (W(60).EQ.0.) GO TO 25                                         HAML039
      NR=NR+1                                                           HAML040
      DRDT(NR)=-SQRT(PHPKR**2+PHPKTH**2+PHPKPH**2)/PHPOM    /C          HAML041
C********* OTHER CALCULATIONS                                          HAML042
   25 CONTINUE                                                          HAML043
      RETURN                                                            HAML044
         END                                                            HAML045-
```

APPENDIX 2. VERSIONS OF THE REFRACTIVE INDEX SUBROUTINE (RINDEX)

This ray tracing program gains versatility without sacrificing speed by having several versions of some of the subroutines. For example, the 8 versions of the refractive index subroutine allow the user to decide for each ray path calculation whether to include or ignore various aspects of the propagation medium such as the earth's magnetic field or collisions between electrons and neutral air molecules.

If collisions are included, the user has the option of using the Appleton-Hartree formula (which assumes a constant collision frequency) or the Sen-Wyller formula (which assumes a Maxwell distribution of electron energies and a collision frequency proportional to energy). The Sen-Wyller formula is generally assumed to be more accurate, especially in the lower ionosphere, but the Appleton-Hartree formula can often be used with an effective collision frequency profile to save computer time.

When the effect of the earth's magnetic field is included and ray paths are calculated near vertical incidence, a spitze (Davies, 1965, p. 202) often occurs in the ray path. (At a spitze, the usual formulas for refractive index become indeterminate because the wave normal is parallel with the earth's magnetic field and the wave frequency equals the local plasma frequency.) Two versions of the refractive index subroutine have been developed to calculate ray paths through a spitze. These two versions will also work in the absence of a spitze, but the standard versions are much faster.

The input to the refractive index subroutines is through blank common and common blocks /XX/, /YY/, and /ZZ/. Output is through common block /RIN/. The refractive index subroutine is called through the entry RINDEX. The subroutine names are used only for user identification. The following 8 versions of the refractive index subroutine are

listed in this appendix:

a.	Subroutine AHWFWC (Appleton-Hartree formula with field, with collisions)	93
b.	Subroutine AHWFNC (Appleton-Hartree formula with field, no collisions)	94
c.	Subroutine AHNFWC (Appleton-Hartree formula no field, with collisions)	96
d.	Subroutine AHNFNC (Appleton-Hartree formula no field, no collisions)	97
e.	Subroutine BQWFWC (Booker Quartic with field, with collisions)	98
f.	Subroutine BQWFNC (Booker Quartic with field, no collisions)	100
g.	Subroutine SWWF (Sen-Wyller formula with field)	102
h.	Subroutine SWNF (Sen-Wyller no field)	105
	Subroutine FGSW	106
	Subroutine FSW	106
	Fresnel integral function C	108
	Fresnel integral function S	108

```
      SUBROUTINE AHWFWC                                                 WFWC001
C          CALCULATES THE REFRACTIVE INDEX AND ITS GRADIENT USING THE   WFWC002
C          APPLETON-HARTREE FORMULA WITH FIELD, WITH COLLISIONS         WFWC003
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RADIAN,K,C,LOGTEN                WFWC004
      COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,KAY2,H,PHPT,PHPR,PHPTH,   WFWC005
     1             PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,KPHPK,POLAR,LPOLAR   WFWC006
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  WFWC007
      COMMON /YY/ MODY,Y,PYPR,PYPTH,PYPPH,YR,PYRPR,PYRPT,PYRPP,YTH,PYTPRWFWC008
     1            ,PYTPT,PYTPP,YPH,PYPPR,PYPPT,PYPPP                    WFWC009
      COMMON /ZZ/ MODZ,Z,PZPR,PZPTH,PZPPH                               WFWC010
      COMMON R,TH,PH,KR,KTH,KPH    /WW/ ID(10),W0,W(400)                WFWC011
      COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART       WFWC012
      EQUIVALENCE (RAY,W(1)),(F,W(6))                                   WFWC013
      LOGICAL SPACE                                                     WFWC014
      REAL KR,KTH,KPH,K2                                                WFWC015
      COMPLEX N2,PNPR,PNPTH,PNPPH,PNPVR,PNPVTH,PNPVPH,NNP,PNPT,         WFWC016
     1        POLAR,LPOLAR,I,U,RAD,D,PNPPS,PNPX,PNPY,PNPZ,UX,UX2,D2,    WFWC017
     2        KAY2,H,PHPT,PHPR,PHPTH,PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,   WFWC018
     3        KPHPK                                                     WFWC019
      DATA (MODRIN=8HAPPLETON,8H-HARTREE,8H FORMULA),(COLL=1.),         WFWC020
     1     (FIELD=1.),                                                  WFWC021
     2     (X=0.),(PXPR=0.),(PXPTH=0.),(PXPPH=0.),(PXPT=0.),            WFWC022
     3     (Y=0.),(PYPR=0.),(PYPTH=0.),(PYPPH=0.),(YR=0.),(PYRPR=0.),   WFWC023
     4     (PYRPT=0.),(PYRPP=0.),(YTH=0.),(PYTPR=0.),(PYTPT=0.),        WFWC024
     5     (PYTPP=0.),(YPH=0.),(PYPPR=0.),(PYPPT=0.),(PYPPP=0.)         WFWC025
     6    ,(Z=0.),(PZPR=0.),(PZPTH=0.),(PZPPH=0.),                      WFWC026
     7     (I=(0.,1.)),(ABSLIM=1.E-5)                                   WFWC027
      ENTRY RINDEX                                                      WFWC028
      OM=PIT2*1.E6*F                                                    WFWC029
      C2=C*C                                                            WFWC030
      K2=KR*KR+KTH*KTH+KPH*KPH                                          WFWC031
      OM2=OM*OM                                                         WFWC032
      VR =C/OM*KR                                                       WFWC033
      VTH=C/OM*KTH                                                      WFWC034
      VPH=C/OM*KPH                                                      WFWC035
      CALL ELECTX                                                       WFWC036
      CALL MAGY                                                         WFWC037
      V2=VR**2+VTH**2+VPH**2                                            WFWC038
      VDOTY=VR*YR+VTH*YTH+VPH*YPH                                       WFWC039
      YLV=VDOTY/V2                                                      WFWC040
      YL2=VDOTY**2/V2                                                   WFWC041
      YT2=Y**2-YL2                                                      WFWC042
      YT4=YT2*YT2                                                       WFWC043
      CALL COLFRZ                                                       WFWC044
      U=CMPLX(1.,-Z)                                                    WFWC045
      UX=U-X                                                            WFWC046
      UX2=UX*UX                                                         WFWC047
      RAD=RAY*CSQRT(YT4+4.*YL2*UX2)                                     WFWC048
      D=2.*U*UX-YT2+RAD                                                 WFWC049
      D2=D*D                                                            WFWC050
      N2=1.-2.*X*UX/D                                                   WFWC051
      PNPPS=2.*X*UX*(-1.+(YT2-2.*UX2)/RAD)/D2                           WFWC052
      PPSPR =YL2/Y*PYPR -(VR*PYRPR+VTH*PYTPR+VPH*PYPPR)*YLV             WFWC053
      PPSPTH=YL2/Y*PYPTH-(VR*PYRPT+VTH*PYTPT+VPH*PYPPT)*YLV             WFWC054
      PPSPPH=YL2/Y*PYPPH-(VR*PYRPP+VTH*PYTPP+VPH*PYPPP)*YLV             WFWC055
      PNPX=-(2.*U*UX2-YT2*(U-2.*X)+(YT4*(U-2.*X)+4.*YL2*UX*UX2)/RAD)/D2 WFWC056
      PNPY=2.*X*UX*(-YT2+(YT4+2.*YL2*UX2)/RAD)/(D2*Y)                   WFWC057
      PNPZ=I*K*(-2.*UX2-YT2+YT4/RAD)/D2                                 WFWC058
      PNPR =PNPX*PXPR +PNPY*PYPR +PNPZ*PZPR +PNPPS*PPSPR                WFWC059
      PNPTH=PNPX*PXPTH+PNPY*PYPTH+PNPZ*PZPTH+PNPPS*PPSPTH               WFWC060
      PNPPH=PNPX*PXPPH+PNPY*PYPPH+PNPZ*PZPPH+PNPPS*PPSPPH               WFWC061
      PNPVR =PNPPS*(VR *YL2/V2-YLV*YR )                                 WFWC062
      PNPVTH=PNPPS*(VTH*YL2/V2-YLV*YTH)                                 WFWC063
      PNPVPH=PNPPS*(VPH*YL2/V2-YLV*YPH)                                 WFWC064
      NNP=N2-(2.*X*PNPX+Y*PNPY+Z*PNPZ)                                  WFWC065
```

```
      PNPT=PNPX*PXPT                                                    WFWC066
      SPACE=REAL(N2).EQ.1..AND.ABS(AIMAG(N2)).LT.ABSLIM                 WFWC067
      POLAR=-I*SQRT(V2)*(-YT2+RAD)/(2.*VOOTY*UX)                        WFWC068
      GAM=(-YT2+RAD)/(2.*UX)                                            WFWC069
      LPOLAR=I*X*SQRT(YT2)/(UX*(U+GAM))                                 WFWC070
      KAY2=OM2/C2*N2                                                    WFWC071
      IF(RSTART.EQ.0.) GO TO 1                                          WFWC072
      SCALE=SQRT(REAL(KAY2)/K2)                                         WFWC073
      KR =SCALE*KR                                                      WFWC074
      KTH=SCALE*KTH                                                     WFWC075
      KPH=SCALE*KPH                                                     WFWC076
    1 CONTINUE                                                          WFWC077
C********* CALCULATES A HAMILTONIAN H                                   WFWC078
      H=.5*(C2*K2/OM2-N2)                                               WFWC079
C********* AND ITS PARTIAL DERIVATIVES WITH RESPECT TO                  WFWC080
C********* TIME, R, THETA, PHI, OMEGA, KR, KTHETA, AND KPHI.            WFWC081
      PHPT =-PNPT                                                       WFWC082
      PHPR =-PNPR                                                       WFWC083
      PHPTH=-PNPTH                                                      WFWC084
      PHPPH=-PNPPH                                                      WFWC085
      PHPOM=-NNP/OM                                                     WFWC086
      PHPKR =C2/OM2*KR -C/OM*PNPVR                                      WFWC087
      PHPKTH=C2/OM2*KTH-C/OM*PNPVTH                                     WFWC088
      PHPKPH=C2/OM2*KPH-C/OM*PNPVPH                                     WFWC089
      KPHPK=N2                                                          WFWC090
      RETURN                                                            WFWC091
         END                                                            WFWC092


      SUBROUTINE AHWFNC                                                 WFNC001
C          CALCULATES THE REFRACTIVE INDEX AND ITS GRADIENT USING THE   WFNC002
C          APPLETON-HARTREE FORMULA WITH FIELD, NO COLLISIONS           WFNC003
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RADIAN,K,C,LOGTEN                WFNC004
      COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,KAY2,KAY2I,               WFNC005
     1              H,HI,PHPT,PHPTI,PHPR,PHPRI,PHPTH,PHPTHI,PHPPH,PHPPHIWFNC006
     2            ,PHPOM,PHPOMI,PHPKR,PHPKRI,PHPKTH,PHPKTI,PHPKPH,PHPKPIWFNC007
     3            ,KPHPK,KPHPKI,POLAR,POLARI,LPOLAR,LPOLRI,SGN         WFNC008
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  WFNC009
      COMMON /YY/ MODY,Y,PYPR,PYPTH,PYPPH,YR,PYRPR,PYRPT,PYRPP,YTH,PYTPRWFNC010
     1           ,PYTPT,PYTPP,YPH,PYPPR,PYPPT,PYPPP                     WFNC011
      COMMON /ZZ/ MODZ,Z(4)                                             WFNC012
      COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART       WFNC013
      COMMON R,TH,PH,KR,KTH,KPH    /WW/ ID(10),W0,W(400)                WFNC014
      EQUIVALENCE (RAY,W(1)),(F,W(6))                                   WFNC015
      LOGICAL SPACE                                                     WFNC016
      REAL KR,KTH,KPH,K2,KPHPK,KPHPKI,KAY2,KAY2I,N2,NNP,LPOLAR,LPOLRI   WFNC017
      DATA (MODRIN=8HAPPLETON,8H-HARTREE,8H FORMULA),(COLL=0.),         WFNC018
     1     (FIELD=1.),(KAY2I=0.),(HI=0.),(PHPTI=0.),(PHPRI=0.),         WFNC019
     2     (PHPTHI=0.),(PHPPHI=0.),(PHPOMI=0.),(PHPKRI=0.),(PHPKTI=0.), WFNC020
     3     (PHPKPI=0.),(KPHPKI=0.),(POLAR=0.),(LPOLAR=0.),              WFNC021
     4     (X=0.),(PXPR=0.),(PXPTH=0.),(PXPPH=0.),(PXPT=0.),            WFNC022
     5     (Y=0.),(PYPR=0.),(PYPTH=0.),(PYPPH=0.),(YR=0.),(PYRPR=0.),   WFNC023
     6     (PYRPT=0.),(PYRPP=0.),(YTH=0.),(PYTPR=0.),(PYTPT=0.),        WFNC024
     7     (PYTPP=0.),(YPH=0.),(PYPPR=0.),(PYPPT=0.),(PYPPP=0.),        WFNC025
     8     (MODZ=1H ),(U=1.)                                            WFNC026
      ENTRY RINDEX                                                      WFNC027
      OM=PIT2*1.E6*F                                                    WFNC028
      C2=C*C                                                            WFNC029
      K2=KR*KR+KTH*KTH+KPH*KPH                                          WFNC030
      OM2=OM*OM                                                         WFNC031
      VR =C/OM*KR                                                       WFNC032
      VTH=C/OM*KTH                                                      WFNC033
      VPH=C/OM*KPH                                                      WFNC034
```

```
      CALL ELECTX                                                       WFNC035
      CALL MAGY                                                         WFNC036
      V2=VR**2+VTH**2+VPH**2                                            WFNC037
      VDOTY=VR*YR+VTH*YTH+VPH*YPH                                       WFNC038
      YLV=VDOTY/V2                                                      WFNC039
      YL2=VDOTY**2/V2                                                   WFNC040
      YT2=Y**2-YL2                                                      WFNC041
      YT4=YT2*YT2                                                       WFNC042
      UX=U-X                                                            WFNC043
      UX2=UX*UX                                                         WFNC044
      RAD=RAY*SQRT(YT4+4.*YL2*UX2)                                      WFNC045
      D=2.*UX-YT2+RAD                                                   WFNC046
      D2=D*D                                                            WFNC047
      N2=1.-2.*X*UX/D                                                   WFNC048
      PNPPS=2.*X*UX*(-1.+(YT2-2.*UX2)/RAD)/D2                           WFNC049
      PPSPR= YL2/Y*PYPR -(VR*PYRPR+VTH*PYTPR+VPH*PYPPR)*YLV             WFNC050
      PPSPTH=YL2/Y*PYPTH-(VR*PYRPT+VTH*PYTPT+VPH*PYPPT)*YLV             WFNC051
      PPSPPH=YL2/Y*PYPPH-(VR*PYRPP+VTH*PYTPP+VPH*PYPPP)*YLV             WFNC052
      PNPX=-(2.*UX2-YT2*(U-2.*X)+(YT4*(U-2.*X)+4.*YL2*UX*UX2)/RAD)/D2   WFNC053
      PNPY=2.*X*UX*(-YT2+(YT4+2.*YL2*UX2)/RAD)/(D2*Y)                   WFNC054
      NNP=N2-(2.*X*PNPX+Y*PNPY)                                         WFNC055
      PNPR =PNPX*PXPR +PNPY*PYPR +PNPPS*PPSPR                           WFNC056
      PNPTH=PNPX*PXPTH+PNPY*PYPTH+PNPPS*PPSPTH                          WFNC057
      PNPPH=PNPX*PXPPH+PNPY*PYPPH+PNPPS*PPSPPH                          WFNC058
      PNPVR =PNPPS*(VR *YL2-VDOTY*YR )/V2                               WFNC059
      PNPVTH=PNPPS*(VTH*YL2-VDOTY*YTH)/V2                               WFNC060
      PNPVPH=PNPPS*(VPH*YL2-VDOTY*YPH)/V2                               WFNC061
      PNPT=PNPX*PXPT                                                    WFNC062
      SPACE=N2.EQ.1.                                                    WFNC063
      POLARI=SQRT(V2)*(YT2-RAD)/(2.*VDOTY*UX)                           WFNC064
      GAM=(-YT2+RAD)/(2.*UX)                                            WFNC065
      LPOLRI=X*SQRT(YT2)/(UX*(U+GAM))                                   WFNC066
      KAY2=OM2/C2*N2                                                    WFNC067
      IF(RSTART.EQ.0.) GO TO 1                                          WFNC068
      SCALE=SQRT(KAY2/K2)                                               WFNC069
      KR =SCALE*KR                                                      WFNC070
      KTH=SCALE*KTH                                                     WFNC071
      KPH=SCALE*KPH                                                     WFNC072
    1 CONTINUE                                                          WFNC073
C********* CALCULATES A HAMILTONIAN H                                   WFNC074
      H=.5*(C2*K2/OM2-N2)                                               WFNC075
C********* AND ITS PARTIAL DERIVATIVES WITH RESPECT TO                  WFNC076
C********* TIME, R, THETA, PHI, OMEGA, KR, KTHETA, AND KPHI.            WFNC077
      PHPT =-PNPT                                                       WFNC078
      PHPR =-PNPR                                                       WFNC079
      PHPTH=-PNPTH                                                      WFNC080
      PHPPH=-PNPPH                                                      WFNC081
      PHPOM=-NNP/OM                                                     WFNC082
      PHPKR =C2/OM2*KR -C/OM*PNPVR                                      WFNC083
      PHPKTH=C2/OM2*KTH-C/OM*PNPVTH                                     WFNC084
      PHPKPH=C2/OM2*KPH-C/OM*PNPVPH                                     WFNC085
      KPHPK=N2                                                          WFNC086
      RETURN                                                            WFNC087
         END                                                            WFNC088-
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```
      SUBROUTINE AHNFWC                                                 NFWC010
C          CALCULATES THE REFRACTIVE INDEX AND ITS GRADIENT USING THE   NFWC011
C          APPLETON-HARTREE FORMULA -- NO FIELD, WITH COLLISIONS        NFWC012
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RADIAN,K,C,LOGTEN                NFWC013
      COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,KAY2,H,PHPT,PHPR,PHPTH,   NFWC014
     1             PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,KPHPK,POLAR,LPOLAR,  NFWC015
     2             SGN                                                  NFWC016
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  NFWC017
      COMMON /YY/ MODY,Y(16)                                            NFWC018
      COMMON /ZZ/ MODZ,Z,PZPR,PZPTH,PZPPH                               NFWC019
      COMMON /RC/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART       NFWC020
      COMMON R,TH,PH,KR,KTH,KPH    /WW/ ID(10),W0,W(400)                NFWC021
      EQUIVALENCE (RAY,W(1)),(F,W(6))                                   NFWC022
      LOGICAL SPACE                                                     NFWC023
      REAL KR,KTH,KPH,K2                                                NFWC024
      COMPLEX KAY2,H,PHPT,PHPR,PHPTH,PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,   NFWC025
     1        KPHPK,POLAR,LPOLAR,U,I,PNPX,PNPZ,                         NFWC026
     2        N2,PNPR,PNPTH,PNPPH,PNPVR,PNPVTH,PNPVPH,NNP,PNPT          NFWC027
      DATA (MODRIN=8HAPPLETON,8H-HARTREE,8H FORMULA),(COLL=1.),         NFWC028
     1     (FIELD=0.),(POLAR=(0.,1.)),(LPOLAR=(0.,0.)),                 NFWC029
     2     (X=0.),(PXPR=0.),(PXPTH=0.),(PXPPH=0.),(PXPT=0.),            NFWC030
     3     (MODY=1H ),                                                  NFWC031
     4     (Z=0.),(PZPR=0.),(PZPTH=0.),(PZPPH=0.),                      NFWC032
     5     (I=(0.,1.)),(ABSLIM=1.E-5),(PNPVR=0.),(PNPVTH=0.),(PNPVPH=0.)NFWC033
      ENTRY RINDEX                                                      NFWC034
      OM=PIT2*1.E6*F                                                    NFWC035
      C2=C*C                                                            NFWC036
      K2=KR*KR+KTH*KTH+KPH*KPH                                          NFWC037
      OM2=OM*OM                                                         NFWC038
      VR =C/OM*KR                                                       NFWC039
      VTH=C/OM*KTH                                                      NFWC040
      VPH=C/OM*KPH                                                      NFWC041
      CALL ELECTX                                                       NFWC042
      CALL COLFRZ                                                       NFWC043
      U=1.-I*Z                                                          NFWC044
      N2=1.-X/U                                                         NFWC045
      PNPX=-1./(2.*U)                                                   NFWC046
      PNPZ=-I*X/(2.*U**2)                                               NFWC047
      NNP=N2-(2.*X*PNPX+Z*PNPZ)                                         NFWC048
      PNPR =PNPX*PXPR +PNPZ*PZPR                                        NFWC049
      PNPTH=PNPX*PXPTH+PNPZ*PZPTH                                       NFWC050
      PNPPH=PNPX*PXPPH+PNPZ*PZPPH                                       NFWC051
      PNPT=PNPX*PXPT                                                    NFWC052
      SPACE=REAL(N2).EQ.1..AND.ABS(AIMAG(N2)).LT.ABSLIM                 NFWC053
      KAY2=OM2/C2*N2                                                    NFWC054
      IF(RSTART.EQ.0.) GO TO 1                                          NFWC055
      SCALE=SQRT(REAL(KAY2)/K2)                                         NFWC056
      KR =SCALE*KR                                                      NFWC057
      KTH=SCALE*KTH                                                     NFWC058
      KPH=SCALE*KPH                                                     NFWC059
    1 CONTINUE                                                          NFWC060
C********* CALCULATES A HAMILTONIAN H                                   NFWC061
      H=.5*(C2*K2/OM2-N2)                                               NFWC062
C********* AND ITS PARTIAL DERIVATIVES WITH RESPECT TO                  NFWC063
C********* TIME, R, THETA, PHI, OMEGA, KR, KTHETA, AND KPHI.            NFWC064
      PHPT =-PNPT                                                       NFWC065
      PHPR =-PNPR                                                       NFWC066
      PHPTH=-PNPTH                                                      NFWC067
      PHPPH=-PNPPH                                                      NFWC068
      PHPOM=-NNP/OM                                                     NFWC069
      PHPKR =C2/OM2*KR                                                  NFWC070
      PHPKTH=C2/OM2*KTH                                                 NFWC071
      PHPKPH=C2/OM2*KPH                                                 NFWC072
      KPHPK=N2                                                          NFWC073
      RETURN                                                            NFWC074
        END                                                             NFWC075-
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```
      SUBROUTINE AHNFNC                                                 NFNC001
C          CALCULATES THE REFRACTIVE INDEX AND ITS GRADIENT USING THE   NFNC002
C          APPLETON-HARTREE FORMULA -- NO FIELD, NO COLLISIONS          NFNC003
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RADIAN,K,C,LOGTEN                NFNC004
      COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,KAY2,KAY2I,               NFNC005
     1             H,HI,PHPT,PHPTI,PHPR,PHPRI,PHPTH,PHPTHI,PHPPH,PHPPHI,NFNC006
     2             PHPOM,PHPOMI,PHPKR,PHPKRI,PHPKTH,PHPKTI,PHPKPH,PHPKPINFNC007
     3            ,KPHPK,KPHPKI,POLAR,POLARI,LPOLAR,LPOLRI,SGN          NFNC008
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  NFNC009
      COMMON /YY/ MODY,Y(16)  /ZZ/ MODZ,Z(4)                            NFNC010
      COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART       NFNC011
      COMMON R,TH,PH,KR,KTH,KPH   /WW/ ID(10),W0,W(400)                 NFNC012
      EQUIVALENCE (RAY,W(1)),(F,W(6))                                   NFNC013
      LOGICAL SPACE                                                     NFNC014
      REAL N2,NNP,KR,KTH,KPH,K2,KPHPK,KPHPKI,KAY2,KAY2I,LPOLAR,LPOLRI   NFNC015
      DATA (MODRIN=8HAPPLETON,8H-HARTREE,8H FORMULA),(COLL=0.),         NFNC016
     1     (FIELD=0.),(KAY2I=0.),(HI=0.),(PHPTI=0.),(PHPRI=0.),         NFNC017
     2     (PHPTHI=0.),(PHPPHI=0.),(PHPOMI=0.),(PHPKRI=0.),(PHPKTI=0.), NFNC018
     3     (PHPKPI=0.),(KPHPKI=0.),(POLAR=0.),(POLARI=1.),(LPOLAR=0.),  NFNC019
     4     (LPOLRI=1.),                                                 NFNC020
     4     (X=0.),(PXPR=0.),(PXPTH=0.),(PXPPH=0.),(PXPT=0.),            NFNC021
     5     (MODY=1H ),(MODZ=1H ),                                       NFNC022
     6     (NNP=1.),(PNPX=-0.5),(PNPVR=0.),(PNPVTH=0.),(PNPVPH=0.)      NFNC023
      ENTRY RINDEX                                                      NFNC024
      OM=PIT2*1.E6*F                                                    NFNC025
      C2=C*C                                                            NFNC026
      K2=KR*KR+KTH*KTH+KPH*KPH                                          NFNC027
      OM2=OM*OM                                                         NFNC028
      VR =C/OM*KR                                                       NFNC029
      VTH=C/OM*KTH                                                      NFNC030
      VPH=C/OM*KPH                                                      NFNC031
      CALL ELECTX                                                       NFNC032
      PNPR =PNPX*PXPR                                                   NFNC033
      PNPTH=PNPX*PXPTH                                                  NFNC034
      PNPPH=PNPX*PXPPH                                                  NFNC035
      PNPT=PNPX*PXPT                                                    NFNC036
      N2=1.-X                                                           NFNC037
      SPACE=N2.EQ.1.                                                    NFNC038
      KAY2=OM2/C2*N2                                                    NFNC039
      IF(RSTART.EQ.0.) GO TO 1                                          NFNC040
      SCALE=SQRT(KAY2/K2)                                               NFNC041
      KR =SCALE*KR                                                      NFNC042
      KTH=SCALE*KTH                                                     NFNC043
      KPH=SCALE*KPH                                                     NFNC044
    1 CONTINUE                                                          NFNC045
C********* CALCULATES A HAMILTONIAN H                                   NFNC046
      H=.5*(C2*K2/OM2-N2)                                               NFNC047
C********* AND ITS PARTIAL DERIVATIVES WITH RESPECT TO                  NFNC048
C********* TIME, R, THETA, PHI, OMEGA, KR, KTHETA, AND KPHI.            NFNC049
      PHPT =-PNPT                                                       NFNC050
      PHPR =-PNPR                                                       NFNC051
      PHPTH=-PNPTH                                                      NFNC052
      PHPPH=-PNPPH                                                      NFNC053
      PHPOM=-NNP/OM                                                     NFNC054
      PHPKR =C2/OM2*KR                                                  NFNC055
      PHPKTH=C2/OM2*KTH                                                 NFNC056
      PHPKPH=C2/OM2*KPH                                                 NFNC057
      KPHPK=N2                                                          NFNC058
      RETURN                                                            NFNC059
         END                                                            NFNC060
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      SUBROUTINE BQWFWC                                                 BQWC001
C********* CALCULATES A HAMILTONIAN H                                   BQWC002
C********* (= BOOKER QUARTIC FOR VERTICAL INCIDENCE, S=0, C=1)          BQWC003
C********* AND ITS PARTIAL DERIVATIVES WITH RESPECT TO                  BQWC004
C********* TIME, R, THETA, PHI, OMEGA, KR, KTHETA, AND KPHI.            BQWC005
C********* WITH FIELD, WITH COLLISIONS                                  BQWC006
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RADIAN,K,C,LOGTEN                BQWC007
      COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,KAY2,H,PHPT,PHPR,PHPTH,   BQWC008
     1             PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,KPHPK,POLAR,LPOLAR,  BQWC009
     2             SGN                                                  BQWC010
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  BQWC011
      COMMON /YY/ MODY,Y,PYPR,PYPTH,PYPPH,YR,PYRPR,PYRPT,PYRPP,YTH,PYTPRBQWC012
     1           ,PYTPT,PYTPP,YPH,PYPPR,PYPPT,PYPPP                     BQWC013
      COMMON /ZZ/ MODZ,Z,PZPR,PZPTH,PZPPH                               BQWC014
      COMMON R,TH,PH,KR,KTH,KPH    /WW/ ID(10),W0,W(400)                BQWC015
      COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART       BQWC016
      COMMON /FLG/ NTYP,NEWWR,NEWWP,PENET,LINES,IHOP,HPUNCH             BQWC017
      EQUIVALENCE (RAY,W(1)),(F,W(6))                                   BQWC018
      LOGICAL SPACE                                                     BQWC019
      REAL KR,KTH,KPH,K2,KDOTY,K4,KDOTY2                                BQWC020
      COMPLEX KAY2,H,PHPT,PHPR,PHPTH,PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,   BQWC021
     1        POLAR,LPOLAR,I,U,RAD,D,PNPPS,PNPX,PNPY,PNPZ,UX,UX2,D2,    BQWC022
     2        KPHPK,U2,A,B,ALPHA,BETA,GAMMA,PHPX,PHPY2,PHPK2,PHPU,PHPZ, BQWC023
     3        N2,PNPR,PNPTH,PNPPH,PNPVR,PNPVTH,PNPVPH,NNP,PNPT          BQWC024
      DATA (MODRIN=8HBOOKER Q,8HUARTIC, ,8HS=0, C=1),(COLL=1.),         BQWC025
     1      (FIELD=1.),                                                 BQWC026
     2      (X=0.),(PXPR=0.),(PXPTH=0.),(PXPPH=0.),(PXPT=0.),           BQWC027
     3      (Y=0.),(PYPR=0.),(PYPTH=0.),(PYPPH=0.),(YR=0.),(PYRPR=0.),  BQWC028
     4      (PYRPT=0.),(PYRPP=0.),(YTH=0.),(PYTPR=0.),(PYTPT=0.),       BQWC029
     5      (PYTPP=0.),(YPH=0.),(PYPPR=0.),(PYPPT=0.),(PYPPP=0.)        BQWC030
     6     ,(Z=0.),(PZPR=0.),(PZPTH=0.),(PZPPH=0.),                     BQWC031
     7      (I=(0.,1.)),(ABSLIM=1.E-5),(SGN=1.)                         BQWC032
      ENTRY RINDEX                                                      BQWC033
      OM=PIT2*1.E6*F                                                    BQWC034
      C2=C*C                                                            BQWC035
      K2=KR*KR+KTH*KTH+KPH*KPH                                          BQWC036
      OM2=OM*OM                                                         BQWC037
      CALL ELECTX                                                       BQWC038
      IF(X.LT..1) GO TO 2                                               BQWC039
      K4=K2*K2                                                          BQWC040
      OM4=OM2*OM2                                                       BQWC041
      C4=C2*C2                                                          BQWC042
      CALL MAGY                                                         BQWC043
      Y2=Y*Y                                                            BQWC044
      KDOTY=KR*YR+KTH*YTH+KPH*YPH                                       BQWC045
      KDOTY2=KDOTY*KDOTY                                                BQWC046
      CALL COLFRZ                                                       BQWC047
      U=CMPLX(1.,-Z)                                                    BQWC048
      U2=U*U                                                            BQWC049
      UX=U-X                                                            BQWC050
      UX2=UX*UX                                                         BQWC051
      A=UX*U2-U*Y2                                                      BQWC052
      B=-2.*U*UX2+Y2*(2.*U-X)                                           BQWC053
      ALPHA=A*C4*K4+X*KDOTY2*C4*K2                                      BQWC054
      BETA=B*C2*OM2*K2-X*KDOTY2*C2*OM2                                  BQWC055
      GAMMA=(UX2-Y2)*UX*OM4                                             BQWC056
      H=ALPHA+BETA+GAMMA                                                BQWC057
      PHPX=-U2*C4*K4+KDOTY2*C4*K2+(4.*U*UX-Y2)*C2*OM2*K2-KDOTY2*C2*OM2+ BQWC058
     1 (-3.*UX2+Y2)*OM4                                                 BQWC059
      PHPY2=-U*C4*K4+(2.*U-X)*C2*OM2*K2-UX*OM4                          BQWC060
      PHPKY2    =X*C2*(C2*K2-OM2)                                       BQWC061
      PHPU=(2.*U*UX+U2-Y2)*C4*K4+2.*(Y2-UX2-2.*U*UX)*C2*K2*OM2+(3.*UX2  BQWC062
     1 -Y2)*OM4                                                         BQWC063
      PHPZ=-I*PHPU                                                      BQWC064
      PHPK2=2.*A*C4*K2+X*KDOTY2*C4+B*C2*OM2                             BQWC065
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      PHPT=PHPX*PXPT                                                    BQWC066
      PHPR =PHPX*PXPR +PHPY2*2.*Y*PYPR +PHPKY2  *2.*KDOTY*              BQWC067
     1 (KR*PYRPR+KTH*PYTPR+KPH*PYPPR) +PHPZ*PZPR                        BQWC068
      PHPTH=PHPX*PXPTH+PHPY2*2.*Y*PYPTH+PHPKY2  *2.*KDOTY*              BQWC069
     1 (KR*PYRPT+KTH*PYTPT+KPH*PYPPT) +PHPZ*PZPTH                       BQWC070
      PHPPH=PHPX*PXPPH+PHPY2*2.*Y*PYPPH+PHPKY2  *2.*KDOTY*              BQWC071
     1 (KR*PYRPP+KTH*PYTPP+KPH*PYPPP) +PHPZ*PZPPH                       BQWC072
      PHPOM=(2.*BETA+4.*GAMMA)/OM                                       BQWC073
     1 -2.*PHPX*X/OM-2.*PHPY2*Y2/OM-2.*PHPKY2  *KDOTY2/OM -PHPZ*Z/OM    BQWC074
      PHPKR= 2.*PHPK2*KR +2.*KDOTY*PHPKY2  *YR                          BQWC075
      PHPKTH=2.*PHPK2*KTH+2.*KDOTY*PHPKY2  *YTH                         BQWC076
      PHPKPH=2.*PHPK2*KPH+2.*KDOTY*PHPKY2  *YPH                         BQWC077
      KAY2=K2*(-BETA+SGN*RAY*CSQRT(BETA**2-4.*ALPHA*GAMMA))/(2.*ALPHA)  BQWC078
C                                                                       BQWC079
      IF(RSTART.EQ.0.) GO TO 1                                          BQWC080
      SCALE=SQRT((-REAL(BETA)+SGN*RAY*SQRT(REAL(BETA)**2                BQWC081
     1 -4.*REAL(ALPHA)*REAL(GAMMA)))/(2.*REAL(ALPHA)))                  BQWC082
      KR =SCALE*KR                                                      BQWC083
      KTH=SCALE*KTH                                                     BQWC084
      KPH=SCALE*KPH                                                     BQWC085
    1 CONTINUE                                                          BQWC086
C********* THE FOLLOWING 3 CARDS USED FOR RAY TRACING IN COMPLEX SPACE  BQWC087
C     IF(CABS((-BETA-SGN*RAY*CSQRT(BETA**2-4.*ALPHA*GAMMA))/ALPHA-2.).  BQWC088
C    1LT.CABS((-BETA+SGN*RAY*CSQRT(BETA**2-4.*ALPHA*GAMMA))/ALPHA-2.)   BQWC089
C    2 .AND.RSTART.EQ.0.) SGN=-SGN                                      BQWC090
      KPHPK=4.*ALPHA+2.*BETA                                            BQWC091
      SPACE=CABS(C2*KAY2/OM2-1.).LT.ABSLIM                              BQWC092
      POLAR =SQRT(K2)*(U+X*OM2/(C2*KAY2-OM2))/KDOTY*I                   BQWC093
      LPOLR = SQRT(Y2-KDOTY2/K2)/UX*(1.-C2*KAY2/OM2)*I                  BQWC094
      RETURN                                                            BQWC095
C           CALCULATES THE REFRACTIVE INDEX AND ITS GRADIENT USING THE  BQWC096
C           APPLETON-HARTREE FORMULA WITH FIELD, WITH COLLISIONS        BQWC097
    2 CONTINUE                                                          BQWC098
      VR =C/OM*KR                                                       BQWC099
      VTH=C/OM*KTH                                                      BQWC100
      VPH=C/OM*KPH                                                      BQWC101
      CALL MAGY                                                         BQWC102
      V2=VR**2+VTH**2+VPH**2                                            BQWC103
      VDOTY=VR*YR+VTH*YTH+VPH*YPH                                       BQWC104
      YLV=VDOTY/V2                                                      BQWC105
      YL2=VDOTY**2/V2                                                   BQWC106
      YT2=Y**2-YL2                                                      BQWC107
      YT4=YT2*YT2                                                       BQWC108
      CALL COLFRZ                                                       BQWC109
      U=CMPLX(1.,-Z)                                                    BQWC110
      UX=U-X                                                            BQWC111
      UX2=UX*UX                                                         BQWC112
      RAD=SGN*RAY*CSQRT(YT4+4.*YL2*UX2)                                 BQWC113
      D=2.*U*UX-YT2+RAD                                                 BQWC114
      D2=D*D                                                            BQWC115
      N2=1.-2.*X*UX/D                                                   BQWC116
      PNPPS=2.*X*UX*(-1.+(YT2-2.*UX2)/RAD)/D2                           BQWC117
      PPSPR= YL2/Y*PYPR -(VR*PYRPR+VTH*PYTPR+VPH*PYPPR)*YLV             BQWC118
      PPSPTH=YL2/Y*PYPTH-(VR*PYRPT+VTH*PYTPT+VPH*PYPPT)*YLV             BQWC119
      PPSPPH=YL2/Y*PYPPH-(VR*PYRPP+VTH*PYTPP+VPH*PYPPP)*YLV             BQWC120
      PNPX=-(2.*U*UX2-YT2*(U-2.*X)+(YT4*(U-2.*X)+4.*YL2*UX*UX2)/RAD)/D2 BQWC121
      PNPY=2.*X*UX*(-YT2+(YT4+2.*YL2*UX2)/RAD)/(D2*Y)                   BQWC122
      PNPZ=I*X*(-2.*UX2-YT2+YT4/RAD)/D2                                 BQWC123
      NNP=N2-(2.*X*PNPX+Y*PNPY+Z*PNPZ)                                  BQWC124
      PNPR =PNPX*PXPR +PNPY*PYPR +PNPZ*PZPR +PNPPS*PPSPR                BQWC125
      PNPTH=PNPX*PXPTH+PNPY*PYPTH+PNPZ*PZPTH+PNPPS*PPSPTH               BQWC126
      PNPPH=PNPX*PXPPH+PNPY*PYPPH+PNPZ*PZPPH+PNPPS*PPSPPH               BQWC127
      PNPVR =PNPPS*(VR *YL2-VDOTY*YR )/V2                               BQWC128
      PNPVTH=PNPPS*(VTH*YL2-VDOTY*YTH)/V2                               BQWC129
      PNPVPH=PNPPS*(VPH*YL2-VDOTY*YPH)/V2                               BQWC130
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      PNPT=PNPX*PXPT                                                    BQWC131
      SPACE=REAL(N2).EQ.1..AND.ABS(AIMAG(N2)).LT.ABSLIM                 BQWC132
      POLAR=-I*SQRT(V2)*(-YT2+RAD)/(2.*VDOTY*UX)                        BQWC133
      GAM=(-YT2+RAD)/(2.*UX)                                            BQWC134
      LPOLAR=I*X*SQRT(YT2)/(UX*(U+GAM))                                 BQWC135
      KAY2=OM2/C2*N2                                                    BQWC136
      IF(RSTART.EQ.0.) GO TO 3                                          BQWC137
      SCALE=SQRT(REAL(KAY2)/K2)                                         BQWC138
      KR =SCALE*KR                                                      BQWC139
      KTH=SCALE*KTH                                                     BQWC140
      KPH=SCALE*KPH                                                     BQWC141
    3 CONTINUE                                                          BQWC142
      H=.5*(C2*K2/OM2-N2)                                               BQWC143
      PHPT =-PNPT                                                       BQWC144
      PHPR =-PNPR                                                       BQWC145
      PHPTH=-PNPTH                                                      BQWC146
      PHPPH=-PNPPH                                                      BQWC147
      PHPOM=-NNP/OM                                                     BQWC148
      PHPKR =C2/OM2*KR -C/OM*PNPVR                                      BQWC149
      PHPKTH=C2/OM2*KTH-C/OM*PNPVTH                                     BQWC150
      PHPKPH=C2/OM2*KPH-C/OM*PNPVPH                                     BQWC151
      KPHPK=N2                                                          BQWC152
      RETURN                                                            BQWC153
         END                                                            BQWC154

      SUBROUTINE BQWFNC                                                 BQNC001
C********* CALCULATES A HAMILTONIAN H                                   BQNC002
C********* (= BOOKER QUARTIC FOR VERTICAL INCIDENCE, S=0, C=1)          BQNC003
C********* AND ITS PARTIAL DERIVATIVES WITH RESPECT TO                  BQNC004
C********* TIME, R, THETA, PHI, OMEGA, KR, KTHETA, AND KPHI.            BQNC005
C********* WITH FIELD, NO COLLISIONS                                    BQNC006
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RADIAN,K,C,LOGTEN                BQNC007
      COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,KAY2,KAY2I,               BQNC008
     1             H,HI,PHPT,PHPTI,PHPR,PHPRI,PHPTH,PHPTHI,PHPPH,PHPPHI,BQNC009
     2             PHPOM,PHPOMI,PHPKR,PHPKRI,PHPKTH,PHPKTI,PHPKPH,PHPKPIBQNC010
     3            ,KPHPK,KPHPKI,POLAR,POLARI,LPOLAR,LPOLRI,SGN         BQNC011
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  BQNC012
      COMMON /YY/ MODY,Y,PYPR,PYPTH,PYPPH,YR,PYRPR,PYRPT,PYRPP,YTH,PYTPRBQNC013
     1           ,PYTPT,PYTPP,YPH,PYPPR,PYPPT,PYPPP                     BQNC014
      COMMON /ZZ/ MODZ,Z(4)                                             BQNC015
      COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART       BQNC016
      COMMON R,TH,PH,KR,KTH,KPH   /WW/ ID(10),W0,W(400)                 BQNC017
      EQUIVALENCE (RAY,W(1)),(F,W(6))                                   BQNC018
      LOGICAL SPACE                                                     BQNC019
      REAL N2,NNP,LPOLAR,LPOLRI,KR,KTH,KPH,K2,KDOTY,K4,KDOTY2,          BQNC020
     1     KPHPK,KPHPKI,KAY2,KAY2I                                      BQNC021
      DATA (MODRIN=8HBOOKER Q,8HUARTIC, ,8HS=0, C=1),(COLL=0.),         BQNC022
     1     (FIELD=1.),(KAY2I=0.),(HI=0.),(PHPTI=0.),(PHPRI=0.),         BQNC023
     2     (PHPTHI=0.),(PHPPHI=0.),(PHPOMI=0.),(PHPKRI=0.),(PHPKTI=0.), BQNC024
     3     (PHPKPI=0.),(KPHPKI=0.),(POLAR=0.),(LPOLAR=0.),              BQNC025
     4     (X=0.),(PXPR=0.),(PXPTH=0.),(PXPPH=0.),(PXPT=0.),            BQNC026
     5     (Y=0.),(PYPR=0.),(PYPTH=0.),(PYPPH=0.),(YR=0.),(PYRPR=0.),   BQNC027
     6     (PYRPT=0.),(PYRPP=0.),(YTH=0.),(PYTPR=0.),(PYTPT=0.),        BQNC028
     7     (PYTPP=0.),(YPH=0.),(PYPPR=0.),(PYPPT=0.),(PYPPP=0.),        BQNC029
     8     (MODZ=1H ),(U=1.),(U2=1.)                                    BQNC030
      ENTRY RINDEX                                                      BQNC031
      OM=PIT2*1.E6*F                                                    BQNC032
      C2=C*C                                                            BQNC033
      K2=KR*KR+KTH*KTH+KPH*KPH                                          BQNC034
      OM2=OM*OM                                                         BQNC035
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      CALL ELECTX                                                       BQNC036
      IF(X.LT..1) GO TO 2                                               BQNC037
      K4=K2*K2                                                          BQNC038
      OM4=OM2*OM2                                                       BQNC039
      C4=C2*C2                                                          BQNC040
      CALL MAGY                                                         BQNC041
      Y2=Y*Y                                                            BQNC042
      KDOTY=KR*YR+KTH*YTH+KPH*YPH                                       BQNC043
      KDOTY2=KDOTY*KDOTY                                                BQNC044
      UX=U-X                                                            BQNC045
      UX2=UX*UX                                                         BQNC046
      A=UX*U2-U*Y2                                                      BQNC047
      B=-2.*U*UX2+Y2*(2.*U-X)                                           BQNC048
      ALPHA=A*C4*K4+X*KDOTY2*C4*K2                                      BQNC049
      BETA=B*C2*OM2*K2-X*KDOTY2*C2*OM2                                  BQNC050
      GAMMA=(UX2-Y2)*UX*OM4                                             BQNC051
      H=ALPHA+BETA+GAMMA                                                BQNC052
      PHPX=-U2*C4*K4+KDOTY2*C4*K2+(4.*U*UX-Y2)*C2*OM2*K2-KDOTY2*C2*OM2+ BQNC053
     1 (-3.*UX2+Y2)*OM4                                                 BQNC054
      PHPY2=-U*C4*K4+(2.*U-X)*C2*OM2*K2-UX*OM4                          BQNC055
      PHPKY2    =X*C2*(C2*K2-OM2)                                       BQNC056
      PHPK2=2.*A*C4*K2+X*KDOTY2*C4+B*C2*OM2                             BQNC057
      PHPT=PHPX*PXPT                                                    BQNC058
      PHPR =PHPX*PXPR +PHPY2*2.*Y*PYPR +PHPKY2  *2.*KDOTY*              BQNC059
     1 (KR*PYRPR+KTH*PYTPR+KPH*PYPPR)                                   BQNC060
      PHPTH=PHPX*PXPTH+PHPY2*2.*Y*PYPTH+PHPKY2  *2.*KDOTY*              BQNC061
     1 (KR*PYRPT+KTH*PYTPT+KPH*PYPPT)                                   BQNC062
      PHPPH=PHPX*PXPPH+PHPY2*2.*Y*PYPPH+PHPKY2  *2.*KDOTY*              BQNC063
     1 (KR*PYRPP+KTH*PYTPP+KPH*PYPPP)                                   BQNC064
      PHPOM=(2.*BETA+4.*GAMMA)/OM                                       BQNC065
     1 -2.*PHPX*X/OM-2.*PHPY2*Y2/OM-2.*PHPKY2  *KDOTY2/OM               BQNC066
      PHPKR= 2.*PHPK2*KR +2.*KDOTY*PHPKY2  *YR                          BQNC067
      PHPKTH=2.*PHPK2*KTH+2.*KDOTY*PHPKY2  *YTH                         BQNC068
      PHPKPH=2.*PHPK2*KPH+2.*KDOTY*PHPKY2  *YPH                         BQNC069
      KAY2 = K2   *(-BETA+RAY*SQRT(BETA**2-4.*ALPHA*GAMMA))/(2.*ALPHA)  BQNC070
      IF(RSTART.EQ.0.) GO TO 1                                          BQNC071
      SCALE=SQRT(KAY2/K2)                                               BQNC072
      KR =SCALE*KR                                                      BQNC073
      KTH=SCALE*KTH                                                     BQNC074
      KPH=SCALE*KPH                                                     BQNC075
    1 CONTINUE                                                          BQNC076
      KPHPK=4.*ALPHA+2.*BETA                                            BQNC077
      SPACE=KAY2.EQ.OM2/C2                                              BQNC078
      POLARI=SQRT(K2)*(U+X*OM2/(C2*KAY2-OM2))/KDOTY                     BQNC079
      LPOLRI= SQRT(Y2-KDOTY2/K2)/UX*(1.-C2*KAY2/OM2)                    BQNC080
      RETURN                                                            BQNC081
C          CALCULATES THE REFRACTIVE INDEX AND ITS GRADIENT USING THE   BQNC082
C          APPLETON-HARTREE FORMULA WITH FIELD, NO COLLISIONS           BQNC083
    2 CONTINUE                                                          BQNC084
      VR =C/OM*KR                                                       BQNC085
      VTH=C/OM*KTH                                                      BQNC086
      VPH=C/OM*KPH                                                      BQNC087
      CALL MAGY                                                         BQNC088
      V2=VR**2+VTH**2+VPH**2                                            BQNC089
      VDOTY=VR*YR+VTH*YTH+VPH*YPH                                       BQNC090
      YLV=VDOTY/V2                                                      BQNC091
      YL2=VDOTY**2/V2                                                   BQNC092
      YT2=Y**2-YL2                                                      BQNC093
      YT4=YT2*YT2                                                       BQNC094
      JX=U-X                                                            BQNC095
      UX2=UX*UX                                                         BQNC096
      RAD=RAY*SQRT(YT4+4.*YL2*UX2)                                      BQNC097
      D=2.*UX-YT2+RAD                                                   BQNC098
      D2=D*D                                                            BQNC099
      N2=1.-2.*X*UX/D                                                   BQNC100
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      PNPPS=2.*X*UX*(-1.+(YT2-2.*UX2)/RAD)/D2                           BQNC101
      PPSPR= YL2/Y*PYPR  -(VR*PYRPR+VTH*PYTPR+VPH*PYPPR)*YLV            BQNC102
      PPSPTH=YL2/Y*PYPTH-(VR*PYRPT+VTH*PYTPT+VPH*PYPPT)*YLV             BQNC103
      PPSPPH=YL2/Y*PYPPH-(VR*PYRPP+VTH*PYTPP+VPH*PYPPP)*YLV             BQNC104
      PNPX=-(2.*UX2-YT2*(U-2.*X)+(YT4*(U-2.*X)+4.*YL2*UX*UX2)/RAD)/D2   BQNC105
      PNPY=2.*X*UX*(-YT2+(YT4+2.*YL2*UX2)/RAD)/(D2*Y)                   BQNC106
      NNP=N2-(2.*X*PNPX+Y*PNPY)                                         BQNC107
      PNPR =PNPX*PXPR +PNPY*PYPR +PNPPS*PPSPR                           BQNC108
      PNPTH=PNPX*PXPTH+PNPY*PYPTH+PNPPS*PPSPTH                          BQNC109
      PNPPH=PNPX*PXPPH+PNPY*PYPPH+PNPPS*PPSPPH                          BQNC110
      PNPVR =PNPPS*(VR *YL2-VDOTY*YR )/V2                               BQNC111
      PNPVTH=PNPPS*(VTH*YL2-VDOTY*YTH)/V2                               BQNC112
      PNPVPH=PNPPS*(VPH*YL2-VDOTY*YPH)/V2                               BQNC113
      PNPT=PNPX*PXPT                                                    BQNC114
      SPACE=N2.EQ.1.                                                    BQNC115
      POLARI=SQRT(V2)*(YT2-RAD)/(2.*VDOTY*UX)                           BQNC116
      GAM=(-YT2+RAD)/(2.*UX)                                            BQNC117
      LPOLRI=X*SQRT(YT2)/(UX*(J+GAM))                                   BQNC118
      KAY2=OM2/C2*N2                                                    BQNC119
      IF(RSTART.EQ.0.) GO TO 3                                          BQNC120
      SCALE=SQRT(KAY2/K2)                                               BQNC121
      KR =SCALE*KR                                                      BQNC122
      KTH=SCALE*KTH                                                     BQNC123
      KPH=SCALE*KPH                                                     BQNC124
    3 CONTINUE                                                          BQNC125
      H=.5*(C2*K2/OM2-N2)                                               BQNC126
      PHPT =-PNPT                                                       BQNC127
      PHPR =-PNPR                                                       BQNC128
      PHPTH=-PNPTH                                                      BQNC129
      PHPPH=-PNPPH                                                      BQNC130
      PHPOM=-NNP/OM                                                     BQNC131
      PHPKR =C2/OM2*KR -C/OM*PNPVR                                      BQNC132
      PHPKTH=C2/OM2*KTH-C/OM*PNPVTH                                     BQNC133
      PHPKPH=C2/OM2*KPH-C/OM*PNPVPH                                     BQNC134
      KPHPK=N2                                                          BQNC135
      RETURN                                                            BQNC136
         END                                                            BQNC137-
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      SUBROUTINE SWWF                                                   SWWF001
C          CALCULATES THE REFRACTIVE INDEX AND ITS GRADIENT USING THE   SWWF002
C          SEN-WYLLER FORMULA -- WITH FIELD                             SWWF003
C          NEEDS SUBROUTINE FSW AND FUNCTIONS C AND S.                  SWWF004
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RADIAN,K,SEA,LOGTEN              SWWF005
      COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,KAY2,H,PHPT,PHPR,PHPTH,   SWWF006
     1             PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,KPHPK,POLAR,LPOLAR,  SWWF007
     2             SGN                                                  SWWF008
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  SWWF009
      COMMON /YY/ MODY,Y,PYPR,PYPTH,PYPPH,YR,PYRPR,PYRPT,PYRPP,YTH,PYTPRSWWF010
     1           ,PYTPT,PYTPP,YPH,PYPPR,PYPPT,PYPPP                     SWWF011
      COMMON /ZZ/ MODZ,Z,PZPR,PZPTH,PZPPH                               SWWF012
      COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART       SWWF013
      COMMON R,TH,PH,KR,KTH,KPH    /WW/ ID(10),W0,W(400)                SWWF014
      EQUIVALENCE (RAY,W(1)),(F,W(6))                                   SWWF015
      LOGICAL SPACE                                                     SWWF016
      REAL KR,KTH,KPH,K2                                                SWWF017
      COMPLEX KAY2,H,PHPT,PHPR,PHPTH,PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,   SWWF018
     1        KPHPK,POLAR,LPOLAR,I,U,RAD,D,PNPPS,PNPX,PNPY,PNPZ,UX,UX2, SWWF019
     2        ALPHA,BETA,GAMMA,A,B,C,TEMP1,TEMP2,TEMP3,ALPOAL,BEPOBE,   SWWF020
     3        GAPOGA,CB2,N2M1,J2,D2GA,DAL,DBET,DGAM,DADY,DADZ,DBDY,DBDZ,SWWF021
     4        DCDY,DCDZ,DUDZ,DT1DX,DT1DY,DT1DZ,DT1DPS,DT2DX,DT2DY,DT2DZ,SWWF022
     5        DT2DPS,DRADDX,DRADDY,DRADDZ,DRDDPS,DDDX,DDDY,DDDZ,DDDPX,  SWWF023
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     6         UPX,N2,PNPR,PNPTH,PNPPH,PNPVR,PNPVTH,PNPVPH,NNP,PNPT        SWWF024
      DATA (MODRIN=8H       SE,8HN-WYLLER,8H FORMULA),(COLL=1.),            SWWF025
     1      (FIELD=1.),(LPOLAR=(0.,0.)),                                    SWWF026
     2      (X=0.),(PXPR=0.),(PXPTH=0.),(PXPPH=0.),(PXPT=0.),               SWWF027
     3      (Y=0.),(PYPR=0.),(PYPTH=0.),(PYPPH=0.),(YR=0.),(PYRPR=0.),      SWWF028
     4      (PYRPT=0.),(PYRPP=0.),(YTH=0.),(PYTPR=0.),(PYTPT=0.),           SWWF029
     5      (PYTPP=0.),(YPH=0.),(PYPPR=0.),(PYPPT=0.),(PYPPP=0.),           SWWF030
     6      (Z=0.),(PZPR=0.),(PZPTH=0.),(PZPPH=0.),                         SWWF031
     7      (I=(0.,1.)),(ABSLIM=1.E-5)                                      SWWF032
      ENTRY RINDEX                                                          SWWF033
      OM=PIT2*1.E6*F                                                        SWWF034
      C2=SEA*SEA                                                            SWWF035
      K2=KR*KR+KTH*KTH+KPH*KPH                                              SWWF036
      OM2=OM*OM                                                             SWWF037
      VR =SEA/OM*KR                                                         SWWF038
      VTH=SEA/OM*KTH                                                        SWWF039
      VPH=SEA/OM*KPH                                                        SWWF040
      CALL ELECTX                                                           SWWF041
      CALL MAGY                                                             SWWF042
      OPY=1.+Y                                                              SWWF043
      OMY=1.-Y                                                              SWWF044
      CALL COLFRZ                                                           SWWF045
      Z2=Z*Z                                                                SWWF046
      CALL FSW(1./Z,ALPHA,DAL)                                              SWWF047
      ALPOAL=DAL/ALPHA                                                      SWWF048
      CALL FSW(OMY/Z,BETA,DBET)                                             SWWF049
      BEPOBE=DBET/BETA                                                      SWWF050
      CALL FSW(OPY/Z,GAMMA,DGAM)                                            SWWF051
      GAPOGA=DGAM/GAMMA                                                     SWWF052
      U=Z/ALPHA                                                             SWWF053
      DUDZ=(1.+ALPOAL/Z)/ALPHA                                              SWWF054
      U2=U*U                                                                SWWF055
      UX=U-X                                                                SWWF056
      UPX=U+X                                                               SWWF057
      B=ALPHA/BETA                                                          SWWF058
      DBDY=B*BEPOBE/Z                                                       SWWF059
      DBDZ=-B*(ALPOAL-OMY*BEPOBE)/Z2                                        SWWF060
      C=ALPHA/GAMMA                                                         SWWF061
      DCDY=-C*GAPOGA/Z                                                      SWWF062
      DCDZ=-C*(ALPOAL-OPY*GAPOGA)/Z2                                        SWWF063
      A=.5*(B+C)-1.                                                         SWWF064
      DADY=.5*(DBDY+DCDY)                                                   SWWF065
      DADZ=.5*(DBDZ+DCDZ)                                                   SWWF066
      TEMP3=(1.-B*C)*U2+A*U*UPX                                             SWWF067
      V2=      VR**2+VTH**2+VPH**2                                          SWWF068
      VDOTY=VR*YR+VTH*YTH+VPH*YPH                                           SWWF069
      YL2=VDOTY**2/V2                                                       SWWF070
      YT2=Y**2-YL2                                                          SWWF071
      Y2=Y*Y                                                                SWWF072
      S2PSI=YT2/Y2                                                          SWWF073
      C2PSI=YL2/Y2                                                          SWWF074
      UX2=UX*UX                                                             SWWF075
      CB2=(C-B)**2                                                          SWWF076
      TEMP1=TEMP3*S2PSI                                                     SWWF077
      DT1DX= A*U*S2PSI                                                      SWWF078
      DT1DY=(U*UPX*DADY-U2*(B*DCDY+C*DBDY))*S2PSI                           SWWF079
      DT1DZ=(2.*U*DUDZ*(1.-B*C+A)+A*X*DUDZ-U2*(B*DCDZ+C*DBDZ)+U*UPX*DADZSWWF080
     1 )*S2PSI                                                              SWWF081
C             (1/YLYT) D/DPSI(TEMP1)                                        SWWF082
      DT1DPS=2.*TEMP1/YT2                                                   SWWF083
      TEMP2=U2*CB2*JX2*C2PSI                                                SWWF084
      DT2DX=-2.*UX*U2*CB2*C2PSI                                             SWWF085
      DT2DY=2.*U2*UX2*C2PSI*(C-B)*(DCDY-DBDY)                               SWWF086
      DT2DZ=2.*U2*UX2*C2PSI*(C-B)*(DCDZ-DBDZ)+2.*TEMP2*(1./U+1./UX)*DUDZSWWF087
C             (1/YLYT) D/DPSI(TEMP2)                                        SWWF088
      DT2DPS=-2.*TEMP2/YL2                                                  SWWF089
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      RAD=RAY*CSQRT(TEMP1**2+TEMP2)                                     SWWF090
      DRADDX=(TEMP1*DT1DX +.5    *DT2DX )/RAD                           SWWF091
      DRADDY=(TEMP1*DT1DY +    .5*DT2DY )/RAD                           SWWF092
      DRADDZ=(TEMP1*DT1DZ +    .5*DT2DZ )/RAD                           SWWF093
C              (1/YLYT) D/DPSI(RAD)                                     SWWF094
      DRDDPS=(TEMP1*DT1DPS+    .5*DT2DPS)/RAD                           SWWF095
      D=2.*U*UX*(1.+A)-TEMP1+RAD+2.*A*U*X*S2PSI                         SWWF096
      DDDX=-2.*U-DT1DX+DRADDX+2.*A*U*S2PSI                              SWWF097
      DDDY= 2.*U*UX*DADY-DT1DY+DRADDY+2.*U*S2PSI*DADY                   SWWF098
      DDDZ=2.*(1.+A)*DUDZ*(U+UX)+2.*U*UX*DADZ-DT1DZ+DRADDZ+2.*X*S2PSI*  SWWF099
     1 (A*DUDZ+J*DADZ)                                                  SWWF100
C            (1/YLYT) D/DPSI(D)                                         SWWF101
      DDDPS=-DT1DPS+DRDDPS+2.*A*U*X/Y2                                  SWWF102
      N2M1=-2.*X*(UX+U*A*S2PSI)/D                                       SWWF103
      N2=1.+N2M1                                                        SWWF104
C                N D/DX(N)                                              SWWF105
      PNPX=-(UX+U*A*S2PSI)*(1.-X*DDDX/D)/D+X/D                          SWWF106
C                N D/DY(N)                                              SWWF107
      PNPY=-X*U*S2PSI/D*DADY-.5*N2M1/D*DDDY                             SWWF108
C                N D/DZ(N)                                              SWWF109
      PNPZ=-X*(1.+A*S2PSI)/D*DUDZ-X*U*S2PSI/D*DADZ-.5*N2M1/D*DDDZ       SWWF110
C              (N/YLYT) D/DPSI(N)                                       SWWF111
      PNPPS=-X*U*A/(D*Y2)      -.5*N2M1/D*DDDPS                         SWWF112
      YLV=VDOTY/V2                                                      SWWF113
C            (YLYT) D/DR(PSI)                                           SWWF114
      PPSPR=YL2/Y*PYPR-(VR*PYRPR+VTH*PYTPR+VPH*PYPPR)*YLV               SWWF115
C            (YLYT) D/DTHETA(PSI)                                       SWWF116
      PPSPTH=YL2/Y*PYPTH-(VR*PYRPT+VTH*PYTPT+VPH*PYPPT)*YLV             SWWF117
C            (YLYT) D/DPHI(PSI)                                         SWWF118
      PPSPPH=YL2/Y*PYPPH-(VR*PYRPP+VTH*PYTPP+VPH*PYPPP)*YLV             SWWF119
      PNPR=PNPX*PXPR+PNPY*PYPR+PNPZ*PZPR+PNPPS*PPSPR                    SWWF120
      PNPTH=PNPX*PXPTH+PNPY*PYPTH+PNPZ*PZPTH+PNPPS*PPSPTH               SWWF121
      PNPPH=PNPX*PXPPH+PNPY*PYPPH+PNPZ*PZPPH+PNPPS*PPSPPH               SWWF122
      PNPVR=PNPPS*(VR*YL2/V2-YLV*YR)                                    SWWF123
      PNPVTH=PNPPS*(VTH*YL2/V2-YLV*YTH)                                 SWWF124
      PNPVPH=PNPPS*(VPH*YL2/V2-YLV*YPH)                                 SWWF125
      NNP=N2-(2.*X*PNPX+Y*PNPY+Z*PNPZ)                                  SWWF126
      PNPT=PNPX*PXPT                                                    SWWF127
      POLAR=I*(TEMP1-RAD)*Y*SQRT(V2)/(U*UX*(C-B)*VDOTY)                 SWWF128
      COSPSI=VDOTY/(Y*SQRT(V2))                                         SWWF129
      LPOLAR=(.5*I*(C-B)*POLAR+A*COSPSI)*SQRT(S2PSI)/                   SWWF130
     1 (POLAR*(UX*(1.+.5*I*(C-B)*COSPSI*POLAR)+A*(U-X*C2PSI)))          SWWF131
      SPACE=REAL(N2).EQ.1..AND.ABS(AIMAG(N2)).LT.ABSLIM                 SWWF132
      KAY2=OM2/C2*N2                                                    SWWF133
      IF(RSTART.EQ.0.) GO TO 1                                          SWWF134
      SCALE=SQRT(REAL(KAY2)/K2)                                         SWWF135
      KR =SCALE*KR                                                      SWWF136
      KTH=SCALE*KTH                                                     SWWF137
      KPH=SCALE*KPH                                                     SWWF138
  1   CONTINUE                                                          SWWF139
C********* CALCULATES A HAMILTONIAN H                                   SWWF140
      H=.5*(C2*K2/OM2-N2)                                               SWWF141
C********* AND ITS PARTIAL DERIVATIVES WITH RESPECT TO                  SWWF142
C********* TIME, R, THETA, PHI, OMEGA, KR, KTHETA, AND KPHI.            SWWF143
      PHPT =-PNPT                                                       SWWF144
      PHPR =-PNPR                                                       SWWF145
      PHPTH=-PNPTH                                                      SWWF146
      PHPPH=-PNPPH                                                      SWWF147
      PHPOM=-NNP/OM                                                     SWWF148
      PHPKR =C2/OM2*KR -SEA/OM*PNPVR                                    SWWF149
      PHPKTH=C2/OM2*KTH-SEA/OM*PNPVTH                                   SWWF150
      PHPKPH=C2/OM2*KPH-SEA/OM*PNPVPH                                   SWWF151
      KPHPK=N2                                                          SWWF152
      RETURN                                                            SWWF153
         END                                                            SWWF154-
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      SUBROUTINE SWNF                                                   SWNF001
C          CALCULATES THE REFRACTIVE INDEX AND ITS GRADIENT USING THE   SWNF002
C          SEN-WYLLER FORMULA -- NO FIELD                               SWNF003
C          NEEDS SUBROUTINES FGSW AND FSW AND FUNCTIONS C AND S.        SWNF004
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RADIAN,K,C,LOGTEN                SWNF005
      COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,KAY2,H,PHPT,PHPR,PHPTH,   SWNF006
     1             PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,KPHPK,POLAR,LPOLAR,  SWNF007
     2             SGN                                                  SWNF008
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  SWNF009
      COMMON /YY/ MODY,Y(16)                                            SWNF010
      COMMON /ZZ/ MODZ,Z,PZPR,PZPTH,PZPPH                               SWNF011
      COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART       SWNF012
      COMMON R,TH,PH,KR,KTH,KPH   /WW/ ID(10),WO,W(400)                 SWNF013
      EQUIVALENCE (RAY,W(1)),(F,W(6))                                   SWNF014
      LOGICAL SPACE                                                     SWNF015
      REAL KR,KTH,KPH,K2                                                SWNF016
      COMPLEX KAY2,H,PHPT,PHPR,PHPTH,PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,   SWNF017
     1        KPHPK,POLAR,LPOLAR,PNPX,PNPZ,F1,DF,G1,DG1,                SWNF018
     2        N2,PNPR,PNPTH,PNPPH,PNPVR,PNPVTH,PNPVPH,NNP,PNPT          SWNF019
      DATA (MODRIN=8H      SE,8HN-WYLLER,8H FORMULA),(COLL=1.),         SWNF020
     1     (FIELD=0.),(POLAR=(0.,1.)),(LPOLAR=(0.,0.)),                 SWNF021
     2     (X=0.),(PXPR=0.),(PXPTH=0.),(PXPPH=0.),(PXPT=0.),            SWNF022
     3     (MODY=1H ),                                                  SWNF023
     4     (Z=0.),(PZPR=0.),(PZPTH=0.),(PZPPH=0.),                      SWNF024
     5     (ABSLIM=1.E-5),(PNPVR=0.),(PNPVTH=0.),(PNPVPH=0.)            SWNF025
      ENTRY RINDEX                                                      SWNF026
      OM=PIT2*1.E6*F                                                    SWNF027
      C2=C*C                                                            SWNF028
      K2=KR*KR+KTH*KTH+KPH*KPH                                          SWNF029
      OM2=OM*OM                                                         SWNF030
      VR =C/OM*KR                                                       SWNF031
      VTH=C/OM*KTH                                                      SWNF032
      VPH=C/OM*KPH                                                      SWNF033
      CALL ELECTX                                                       SWNF034
      CALL COLFRZ                                                       SWNF035
      CALL FGSW(1./Z,F1,DF1,G1,DG1)                                     SWNF036
      N2=1.-X*G1                                                        SWNF037
      PNPX=-.5*G1                                                       SWNF038
      PNPZ=.5*X*DG1/Z**2                                                SWNF039
      PNPR=PNPX*PXPR+PNPZ*PZPR                                          SWNF040
      PNPTH=PNPX*PXPTH+PNPZ*PZPTH                                       SWNF041
      PNPPH=PNPX*PXPPH+PNPZ*PZPPH                                       SWNF042
      NNP=N2-(2.*X*PNPX+Z*PNPZ)                                         SWNF043
      PNPT=PNPX*PXPT                                                    SWNF044
      SPACE=REAL(N2).EQ.1..AND.ABS(AIMAG(N2)).LT.ABSLIM                 SWNF045
      KAY2=OM2/C2*N2                                                    SWNF046
      IF(RSTART.EQ.0.) GO TO 1                                          SWNF047
      SCALE=SQRT(REAL(KAY2)/K2)                                         SWNF048
      KR =SCALE*KR                                                      SWNF049
      KTH=SCALE*KTH                                                     SWNF050
      KPH=SCALE*KPH                                                     SWNF051
    1 CONTINUE                                                          SWNF052
C********* CALCULATES A HAMILTONIAN H                                   SWNF053
      H=.5*(C2*K2/OM2-N2)                                               SWNF054
C********* AND ITS PARTIAL DERIVATIVES WITH RESPECT TO                  SWNF055
C********* TIME, R, THETA, PHI, OMEGA, KR, KTHETA, AND KPHI.            SWNF056
      PHPT =-PNPT                                                       SWNF057
      PHPR =-PNPR                                                       SWNF058
      PHPTH=-PNPTH                                                      SWNF059
      PHPPH=-PNPPH                                                      SWNF060
      PHPOM=-NNP/OM                                                     SWNF061
      PHPKR =C2/OM2*KR                                                  SWNF062
      PHPKTH=C2/OM2*KTH                                                 SWNF063
      PHPKPH=C2/OM2*KPH                                                 SWNF064
      KPHPK=N2                                                          SWNF065
      RETURN                                                            SWNF066
         END                                                            SWNF067-
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      SUBROUTINE FGSW (X,F,DF,G,DG)                                     FGSW001
      COMPLEX F,DF,G,DG                                                 FGSW002
      CALL FSW (X,F,DF)                                                 FGSW003
      IF(ABS(X).GT.50.) GO TO 1                                         FGSW004
      G=X*F                                                             FGSW005
      DG=F+X*DF                                                         FGSW006
      RETURN                                                            FGSW007
    1 X2=X*X                                                            FGSW008
      X3=X2*X                                                           FGSW009
      T2=2.*X2                                                          FGSW010
      T3=3.*X2                                                          FGSW011
      T4=4.*X2                                                          FGSW012
      T8=8.*X2                                                          FGSW013
      T12=12.*X2                                                        FGSW014
      T16=16.*X2                                                        FGSW015
      G=CMPLX(1.-35./T4*(1.-99./T4*(1.-195./T4*(1.-323./T4)))/T4,       FGSW016
     12.5*(1.-63./T4*(1.-143./T4*(1.-255./T4*(1.-399./T4))))/X)         FGSW017
      DG=.5*CMPLX(35.*(1.-99./T2*(1.-585./T8*(1.-323./T3*(1.-2415./T16))FGSW018
     1 ))/X3,                                                           FGSW019
     2-5.*(1.-189./T4*(1.-715./T12*(1.-357./T4*(1.-513./T4))))/X2)      FGSW020
      RETURN                                                            FGSW021
         END                                                            FGSW 22-
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      SUBROUTINE FSW (Z,F,DF)                                           FSW 001
C        F(Z) = Z*C3/2(Z) + 2.5*I*C5/2(Z)  AND  DF(Z) = DF/DZ           FSW 002
C        WHERE THE INPUT Z IS REAL AND THE OUTPUT F AND DF ARE COMPLEX. FSW 003
C       NEEDS THE SUBPROGRAMS FOR THE FRESNEL INTEGRAL FUNCTIONS S AND CFSW 004
      DIMENSION A(10),B(10),D(10)                                       FSW 005
      COMPLEX F,DF,C1,C2,C3,C8,W,TEMP,I                                 FSW 006
      DATA (I=(0.,1.)),(PI=3.1415926536),(A3=1.333333333)               FSW 007
      DATA (C2=(1.,1.)),(C3=(1.,-1.)),(C4=.79788456  ),(C6=1.333333333) FSW 008
C        C4=SQRT(2./PI)                                                 FSW 009
      DATA(A=.36230845E-02,.29579186E+00,.23193588E+01,.91355870E+01,   FSW 010
     1.25856287E+02,.60488560E+02,.12562218E+03,.24214980E+03,          FSW 011
     2.44918106E+03,.84244774E+03),                                     FSW 012
     3(B=.16747479E-02,.84796280E-01,.25285001E+00,.22665867E+00,       FSW 013
     4.83871933E-01,.13811875E-01,.98017417E-03,.26299148E-04,          FSW 014
     5.19761006E-06,.18781476E-09),                                     FSW 015
     6(D=.10080653E-03,.46117941E-01,.38507643E+00,.68507885E+00,       FSW 016
     7.42648105E+00,.10742102E+00,.10985920E-01,.40924533E-03,          FSW 017
     8.41881263E-05,.54513142E-08),(G=1.5045055)                        FSW 018
      C1=2./3.*I                                                        FSW 019
      C8=C2*A3*SQRT(PI/2.)                                              FSW 020
      X=Z                                                               FSW 021
      X2=X*X                                                            FSW 022
      X3=X2*X                                                           FSW 023
      IF(ABS(X).GT.50.) GO TO 500                                       FSW 024
      IF(ABS(X).GT.6.) GO TO 1                                          FSW 025
      IF(ABS(X).LT..05)GO TO 200                                        FSW 026
C                                FRESNEL                                FSW 027
      IF(X.GT.0.) GO TO 300                                             FSW 028
100    Y=C4*SQRT(-X)                                                    FSW 029
      X2=X*X                                                            FSW 030
      W=(COS(X)+I*SIN(X))*(1.-C3*(C(Y)+I*S(Y)))                         FSW 031
      F  =C1+C6*(X+C3*X*X/Y*W)                                          FSW 032
      DF=A3*CMPLX(1.,X)+CMPLX(1.5,X)*A3*C3*X/Y*W                        FSW 033
      RETURN                                                            FSW 034
300   Y=C4*SQRT(X)                                                      FSW 035
      X2=X*X                                                            FSW 036
      W=(COS(X)+I*SIN(X))*(1.-C2*(C(Y)-I*S(Y)))                         FSW 037
      F  =C1+C6*(X-C2*X*X/Y*W)                                          FSW 038
      DF=A3*CMPLX(1.,X)-CMPLX(1.5,X)*A3*C2*X/Y*W                        FSW 039
      RETURN                                                            FSW 040
```

```
C                                POWER SERIES                                    FSW 041
  200 X=ABS(Z)                                                                   FSW 042
      X2=X*X                                                                     FSW 043
      X3=X2*X                                                                    FSW 044
      X4=X*X3                                                                    FSW 045
      X5=X*X4                                                                    FSW 046
      TEMP=-C8* SQRT(X)*CEXP(I*X)                                                FSW 047
      F=CMPLX(4./3.*X-16./9.*X3+64./315.*X5,2./3.+8./3.*X2-32./45.*X4)           FSW 048
     1   +TEMP*X                                                                 FSW 049
      DF=CMPLX(4./3.-16./3.*X2+64./63.*X4,16./3.*X-128./45.*X3                   FSW 050
     1   +256./945.*X5)                                                          FSW 051
     2   +TEMP*CMPLX(1.5,X)                                                      FSW 052
      IF(Z.GE.0.) RETURN                                                         FSW 053
      F=-CONJG(F)                                                                FSW 054
      DF=CONJG(DF)                                                               FSW 055
      RETURN                                                                     FSW 056
C                                   HERMITE                                      FSW 057
    1 XQ = X**2                                                                  FSW 058
      X2=XQ                                                                      FSW 059
      FR = 0.                                                                    FSW 060
      FI = 0.                                                                    FSW 061
      DFR = 0.                                                                   FSW 062
      DFI = 0.                                                                   FSW 063
      DO 2 J = 1,10                                                              FSW 064
      SS = A(J) + XQ                                                             FSW 065
      SB = B(J)/SS                                                               FSW 066
      SD = D(J)/SS                                                               FSW 067
      FR = FR + SB                                                               FSW 068
      FI = FI + SD                                                               FSW 069
      DFR = DFR + SB/SS                                                          FSW 070
    2 DFI = DFI + SD/SS                                                          FSW 071
      F = CMPLX(X*FR,FI)*G                                                       FSW 072
      DF = G*(FR - 2.*X*CMPLX(X*DFR,DFI))                                        FSW 073
      RETURN                                                                     FSW 074
C                                  ASYMPTOTIC                                    FSW 075
  500 X2=X*X                                                                     FSW 076
      X3=X2*X                                                                    FSW 077
      X4=X3*X                                                                    FSW 078
      X5=X4*X                                                                    FSW 079
      T2=2.*X2                                                                   FSW 080
      T3=3.*X2                                                                   FSW 081
      T4=4.*X2                                                                   FSW 082
      T8=8.*X2                                                                   FSW 083
      T16=16.*X2                                                                 FSW 084
      T28=28.*X2                                                                 FSW 085
      F=CMPLX((1.-35./T4*(1.-99./T4*(1.-195./T4*(1.-323./T4))))/X                FSW 086
     1,5.*(1.-63./T4*(1.-143./T4*(1.-255./T4*(1.-399./T4))))/T2)                 FSW 087
      DF=-CMPLX((1.-105./T4*(1.-165./T4*(1.-273./T4*(1.-2907./T28))))/X2FSW 088
     1,5.*(1.-63./T2*(1.-429./T8*(1.-255./T3*(1.-1995./T16))))/X3)               FSW 089
      RETURN                                                                     FSW 090
         END                                                                     FSW  91-
```

```
      FUNCTION C(X)                                                     C   001
      DOUBLEPRECISION   PIH, XD, Y, V, A, QZ, QN, Q, Z                  C   002
      DATA (A1=0.3183099),(A2=0.10132),(B1=0.0968),(B2=0.154)           C   003
      PIH = 1.570796326794897                                           C   004
      XA = ABS(X)                                                       C   005
      IF (XA.GT.4.)   GOTO 20                                           C   006
C                                                                       C   007
      XD = X                                                            C   008
      Y = PIH*XD*XD                                                     C   009
      V = Y*Y                                                           C   010
      A = 1.D0                                                          C   011
      Z = A                                                             C   012
      M = 15.*(XA + 1.)                                                 C   013
      DO 10   I = 1, M                                                  C   014
      KZ=2*(I-1)                                                        C   015
      KV=4*(I-1)                                                        C   016
      QZ = KV + 1                                                       C   017
      QN = (KZ + 1)*(KZ + 2)*(KV + 5)                                   C   018
       Q = QZ/QN                                                        C   019
      A = -A*Q*V                                                        C   020
   10 Z = Z + A                                                         C   021
      Z = Z*XD                                                          C   022
      C = Z                                                             C   023
      RETURN                                                            C   024
C                                                                       C   025
   20  W = PIH*X*X                                                      C   026
      XV=XA**4                                                          C   027
      C=0.5+(A1-B1/XV)*SIN(W)/XA-(A2-B2/XV)*COS(W)/XA**3                C   028
      IF (X.LT.0.)   C = -C                                             C   029
      RETURN                                                            C   030
         END                                                            C    31-
```

```
      FUNCTION S(X)                                                     S   001
      DOUBLEPRECISION   PIH, XD, Y, V, A, QZ, QN, Q, Z                  S   002
      DATA (A1=0.3183099),(A2=0.10132),(B1=0.0968),(B2=0.154)           S   003
      PIH = 1.570796326794897                                           S   004
C                                                                       S   005
      XA = ABS(X)                                                       S   006
      IF (XA.GT.4.)   GOTO 20                                           S   007
C                                                                       S   008
      XD = X                                                            S   009
      Y = PIH*XD*XD                                                     S   010
      V = Y*Y                                                           S   011
      A = Y/3.D0                                                        S   012
      Z = A                                                             S   013
      M = 15.*(XA + 1.)                                                 S   014
      DO 10   I = 1, M                                                  S   015
      KZ=2*(I-1)                                                        S   016
      KV=4*(I-1)                                                        S   017
      QZ = KV + 3                                                       S   018
      QN = (KZ + 2)*(KZ + 3)*(KV + 7)                                   S   019
       Q = QZ/QN                                                        S   020
       A = -A*Q*V                                                       S   021
   10 Z = Z + A                                                         S   022
      Z = Z*XD                                                          S   023
      S = Z                                                             S   024
      RETURN                                                            S   025
C                                                                       S   026
   20  W = PIH*X*X                                                      S   027
      XV=XA**4                                                          S   028
      S=0.5-(A1-B1/XV)*COS(W)/XA-(A2-B2/XV)*SIN(W)/XA**3                S   029
      IF (X.LT.0.)   S = -S                                             S   030
      RETURN                                                            S   031
         END                                                            S    32-
```

APPENDIX 3. ELECTRON DENSITY SUBROUTINES WITH INPUT PARAMETER FORMS

The following electron density models are available. The input parameter forms, which describe the model, and the subroutine listings are given on the pages shown.

a.	Tabular profiles (TABLEX)	111
b.	Subroutine GAUSEL	113
c.	Chapman layer with tilts, ripples, and gradients (CHAPX)	115
d.	Chapman layer with variable scale height (VCHAPX)	117
e.	Double, tilted α-Chapman layer (DCHAPT)	118
f.	Linear Layer (LINEAR)	120
g.	Plain or quasi-parabolic layer (QPARAB)	121
h.	Analytic equatorial model (BULGE)	122
i.	Exponential profile (EXPX)	124

A further source of versatility in this ray tracing program is the ease with which specific ionospheric models, suited to the users needs, may be introduced. To add electron density models not included in the program, the user must write a subroutine that calculates the normalized electron density (X) and its gradient ($\partial X/\partial r$, $\partial X/\partial\theta$, $\partial X/\partial\varphi$) as a function of position in spherical coordinates (r, θ, φ). ($X = 80.5 \times 10^{-6} N/f^2$, where N is the electron density in cm^{-3} and f is the wave frequency in MHz.)

Both X and its gradient must be continuous functions of position. The formulas for $\partial X/\partial r$, $\partial X/\partial\theta$, and $\partial X/\partial\varphi$ must be consistent with the variation of X with r, θ, and φ. Otherwise, the program will run slowly and give incorrect results.

The coordinates r, θ, φ refer to the computational coordinate system, which may not be the same as geographic coordinates. In particular, they are geomagnetic coordinates when the earth-centered dipole model of the earth's magnetic field is used.

The input to the subroutine (r, θ, ϕ) is through blank common. (See

Table 3.) The output is through common block /XX/. (See Table 8.) It is useful if the name of the subroutine suggests the model to which it corresponds. The subroutine should have an entry point ELECTX so that other subroutines in the program can call it. Any parameters needed by the subroutine should be input into W101 through W149 of the W array. (See Table 2.) If the model needs massive amounts of data, these should be read in by the subroutine following the example of TABLEX. As in the already existing electron density subroutines, provision should be made for perturbations to the electron density model (irregularities) by having the statement

```
IF(PERT.NE.0.) CALL ELECT1
```

before the RETURN statement at the end of the subroutine.

INPUT PARAMETER FORM FOR SUBROUTINE TABLEX

IONOSPHERIC ELECTRON DENSITY PROFILE

First card tells how many profile points in I4 format. The cards following the first card give the height and electron density of the profile points one point per card in F8.2, E12.4 format. The heights must be in increasing order. Set W100 = 1.0 to read in a new profile. After the cards are read, TABLEX will reset W100 = 0.0. This subroutine makes an exponential extrapolation down using the bottom 2 points in the profile.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20		1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20	
HEIGHT h km	ELECTRON DENSITY N ELECTRONS/cm³	HEIGHT h km	ELECTRON DENSITY N ELECTRONS/cm³

```
      SUBROUTINE TABLEX                                                 TABX001
C        CALCULATES ELECTRON DENSITY AND GRADIENT FROM PROFILES HAVING  TABX002
C        THE SAME FORM AS THOSE USED BY CROFTS RAY TRACING PROGRAM      TABX003
C MAKES AN EXPONENTIAL EXTRAPOLATION DOWN USING THE BOTTOM TWO POINTS   TABX004
C        NEEDS SUBROUTINE GAUSEL                                        TABX005
      DIMENSION HPC(250),FN2C(250),ALPHA(250),BETA(250),GAMMA(250),     TABX006
     1 DELTA(250),SLOPE(250),MAT(4,5)                                   TABX007
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RAD,K,DUM(2)                     TABX008
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  TABX009
      COMMON R(6) /WW/ ID(10),W0,W(400)                                 TABX010
      EQUIVALENCE (EARTHR,W(2)),(F,W(6)),(READFN,W(100)),(PERT,W(150))  TABX011
      REAL MAT,K                                                        TABX012
      DATA (MODX(1)=6HTABLEX)                                           TABX013
      ENTRY ELECTX                                                      TABX014
      IF (READFN.EQ.0.) GO TO 10                                        TABX015
      READFN=0.                                                         TABX016
      READ 1000, NOC,(HPC(I),FN2C(I),I=1,NOC)                           TABX017
 1000 FORMAT (I4/(F8.2,E12.4))                                          TABX018
      PRINT 1200, (HPC(I),FN2C(I),I=1,NOC)                              TABX019
 1200 FORMAT(1H1,14X,6HHEIGHT,4X,16HELECTRON DENSITY/(1X,F20.10,E20.10))TABX020
      A=0.                                                              TABX021
      IF(FN2C(1).NE.0.) A=ALOG(FN2C(2)/FN2C(1))/(HPC(2)-HPC(1))         TABX022
      FN2C(1)=K*FN2C(1)                                                 TABX023
      FN2C(2)=K*FN2C(2)                                                 TABX024
      SLOPE(1)=A*FN2C(1)                                                TABX025
      SLOPE(NOC)=0.                                                     TABX026
      NMAX=1                                                            TABX027
      DO 6 I=2,NOC                                                      TABX028
      IF (FN2C(I).GT.FN2C(NMAX)) NMAX=I                                 TABX029
      IF (I.EQ.NOC) GO TO 4                                             TABX030
      FN2C(I+1)=K*FN2C(I+1)                                             TABX031
      DO 3 J=1,3                                                        TABX032
      M=I+J-2                                                           TABX033
      MAT(J,1)=1.                                                       TABX034
      MAT(J,2)=HPC(M)                                                   TABX035
      MAT(J,3)=HPC(M)**2                                                TABX036
    3 MAT(J,4)=FN2C(M)                                                  TABX037
      CALL GAUSEL (MAT,4,3,4,NRANK)                                     TABX038
      IF (NRANK.LT.3) GO TO 60                                          TABX039
      SLOPE(I)=MAT(2,4)+2.*MAT(3,4)*HPC(I)                              TABX040
    4 DO 5 J=1,2                                                        TABX041
      M=I+J-2                                                           TABX042
      MAT(J,1)=1.                                                       TABX043
      MAT(J,2)=HPC(M)                                                   TABX044
      MAT(J,3)=HPC(M)**2                                                TABX045
      MAT(J,4)=HPC(M)**3                                                TABX046
      MAT(J,5)=FN2C(M)                                                  TABX047
      L=J+2                                                             TABX048
      MAT(L,1)=0.                                                       TABX049
      MAT(L,2)=1.                                                       TABX050
      MAT(L,3)=2.*HPC(M)                                                TABX051
      MAT(L,4)=3.*HPC(M)**2                                             TABX052
    5 MAT(L,5)=SLOPE(M)                                                 TABX053
      CALL GAUSEL (MAT,4,4,5,NRANK)                                     TABX054
      IF (NRANK.LT.4) GO TO 60                                          TABX055
      ALPHA(I)=MAT(1,5)                                                 TABX056
      BETA(I)=MAT(2,5)                                                  TABX057
      GAMMA(I)=MAT(3,5)                                                 TABX058
    6 DELTA(I)=MAT(4,5)                                                 TABX059
      HMAX=HPC(NMAX)                                                    TABX060
      NH=2                                                              TABX061
   10 H=R(1)-EARTHR                                                     TABX062
      F2=F*F                                                            TABX063
      PXPR=PXPTH=PXPPH=0.                                               TABX064
      IF (H.GE.HPC(1)) GO TO 12                                         TABX065
   11 NH=2                                                              TABX066
      X=0.                                                              TABX067
```

```
      IF(FN2C(1).EQ.0.) GO TO 50                                        TABX068
      X=FN2C(1)*EXP(A*(H-HPC(1)))/F2                                    TABX069
      PXPR=A*X                                                          TABX070
      GO TO 50                                                          TABX071
   12 IF (H.GE.HPC(NOC)) GO TO 18                                       TABX072
      NSTEP=1                                                           TABX073
      IF (H.LT.HPC(NH-1)) NSTEP=-1                                      TABX074
   15 IF (HPC(NH-1).LE.H.AND.H.LT.HPC(NH)) GO TO 16                     TABX075
      NH=NH+NSTEP                                                       TABX076
      GO TO 15                                                          TABX077
   16 X=(ALPHA(NH)+H*(BETA(NH)+H*(GAMMA(NH)+H*DELTA(NH))))/F2           TABX078
      PXPR=(BETA(NH)+H*(2.*GAMMA(NH)+H*3.*DELTA(NH)))/F2                TABX079
      GO TO 50                                                          TABX080
   18 X=FN2C(NOC)/F2                                                    TABX081
   50 IF (PERT.NE.0.) CALL ELECT1                                       TABX082
      RETURN                                                            TABX083
   60 PRINT 6000, I,HPC(I)                                              TABX084
 6000 FORMAT(4H THE,I4,55HTH POINT IN THE ELECTRON DENSITY PROFILE HAS TTABX085
     1HE HEIGHT,F8.2,40H KM, WHICH IS THE SAME AS ANOTHER POINT.)       TABX086
      CALL EXIT                                                         TABX087
         END                                                            TABX088
```

```
      SUBROUTINE GAUSEL (C,NRD,NRR,NCC,NSF)                             GAUS001
C********* SAME AS SUBROUTINE GAUSSEL WRITTEN BY L. DAVID LEWIS ********GAUS002
      DIMENSION C(NRD,NCC),L(128,2)                                     GAUS003
C          BITS = 2.**-18                                               GAUS004
      DATA (BITS=3.8146972656E-6)                                       GAUS005
      NR=NRR                                                            GAUS006
      NC=NCC                                                            GAUS007
      IF(NC.LT.NR.OR.NR.GT.128.OR.NR.LE.0) CALL EXIT                    GAUS008
C                                                                       GAUS009
C          INITIALIZE.                                                  GAUS010
      NSF=0                                                             GAUS011
      NRM=NR-1                                                          GAUS012
      NRP=NR+1                                                          GAUS013
      D=1.                                                              GAUS014
      LSD=1                                                             GAUS015
      DO 1 KR=1,NR                                                      GAUS016
      L(KR,1)=KR                                                        GAUS017
1     L(KR,2)=0                                                         GAUS018
      IF(NR.EQ.1) GO TO 42                                              GAUS019
C                                                                       GAUS020
C          ELIMINATION PHASE.                                           GAUS021
      DO 41 KP=1,NRM                                                    GAUS022
      KPP=KP+1                                                          GAUS023
      PM=0.                                                             GAUS024
      MPN=0                                                             GAUS025
C                                                                       GAUS026
C          SEARCH COLUMN KP FROM DIAGONAL DOWN FOR MAX PIVOT.           GAUS027
      DO 2 KR=KP,NR                                                     GAUS028
      LKR=L(KR,1)                                                       GAUS029
      PT=ABS(C(LKR,KP))                                                 GAUS030
      IF(PT.LE.PM) GO TO 2                                              GAUS031
      PM=PT                                                             GAUS032
      MPN=KR                                                            GAUS033
      LMP=LKR                                                           GAUS034
2     CONTINUE                                                          GAUS035
C                                                                       GAUS036
C          IF MAX PIVOT IS ZERO, MATRIX IS SINGULAR.                    GAUS037
      IF(MPN.EQ.0) GO TO 9                                              GAUS038
      NSF=NSF+1                                                         GAUS039
      IF(MPN.EQ.KP) GO TO 3                                             GAUS040
```

```
C                                                                       GAUS041
C          NEW ROW NUMBER KP HAS MAX PIVOT.                             GAUS042
      LSD=-LSD                                                          GAUS043
      L(KP,2)=L(MPN,1)=L(KP,1)                                          GAUS044
      L(KP,1)=LMP                                                       GAUS045
C                                                                       GAUS046
C          ROW OPERATIONS TO ZERO COLUMN KP BELOW DIAGONAL.             GAUS047
3     MKP=L(KP,1)                                                       GAUS048
      P=C(MKP,KP)                                                       GAUS049
      D=D*P                                                             GAUS050
      DO 41 KR=KPP,NR                                                   GAUS051
      MKR=L(KR,1)                                                       GAUS052
      Q=C(MKR,KP)/P                                                     GAUS053
      IF(Q.EQ.0.) GO TO 41                                              GAUS054
C                                                                       GAUS055
C          SUBTRACT Q * PIVOT ROW FROM ROW KR.                          GAUS056
      DO 4 LC=KPP,NC                                                    GAUS057
      R=Q*C(MKP,LC)                                                     GAUS058
      C(MKR,LC)=C(MKR,LC)-R                                             GAUS059
4     IF(ABS(C(MKR,LC)).LT.ABS(R)*BITS) C(MKR,LC)=0.                    GAUS060
41    CONTINUE                                                          GAUS061
C                                                                       GAUS062
C          LOWER RIGHT HAND CORNER.                                     GAUS063
42    LNR=L(NR,1)                                                       GAUS064
      P=C(LNR,NR)                                                       GAUS065
      IF(P.EQ.0.) GO TO 9                                               GAUS066
      NSF=NSF+1                                                         GAUS067
      D=D*P*LSD                                                         GAUS068
      IF(NR.EQ.NC) GO TO 8                                              GAUS069
C                                                                       GAUS070
C          BACK SOLUTION PHASE.                                         GAUS071
      DO 61 MC=NRP,NC                                                   GAUS072
      C(LNR,MC)=C(LNR,MC)/P                                             GAUS073
      IF(NR.EQ.1) GO TO 61                                              GAUS074
      DO 6 LL=1,NRM                                                     GAUS075
      KR=NR-LL                                                          GAUS076
      MR=L(KR,1)                                                        GAUS077
      KRP=KR+1                                                          GAUS078
      DO 5 MS=KRP,NR                                                    GAUS079
      LMS=L(MS,1)                                                       GAUS080
      R=C(MR,MS)*C(LMS,MC)                                              GAUS081
      C(MR,MC)=C(MR,MC)-R                                               GAUS082
5     IF(ABS(C(MR,MC)).LT.ABS(R)*BITS) C(MR,MC)=0.                      GAUS083
6     C(MR,MC)=C(MR,MC)/C(MR,KR)                                        GAUS084
61    CONTINUE                                                          GAUS085
C                                                                       GAUS086
C          SHUFFLE SOLUTION ROWS BACK TO NATURAL ORDER.                 GAUS087
      DO 71 LL=1,NRM                                                    GAUS088
      KR=NR-LL                                                          GAUS089
      MKR=L(KR,2)                                                       GAUS090
      IF(MKR.EQ.0) GO TO 71                                             GAUS091
      MKP=L(KR,1)                                                       GAUS092
      DO 7 LC=NRP,NC                                                    GAUS093
      Q=C(MKR,LC)                                                       GAUS094
      C(MKR,LC)=C(MKP,LC)                                               GAUS095
7     C(MKP,LC)=Q                                                       GAUS096
71    CONTINUE                                                          GAUS097
C                                                                       GAUS098
C          NORMAL AND SINGULAR RETURNS.  GOOD SOLUTION COULD HAVE D=0.  GAUS099
8     C(1,1)=D                                                          GAUS100
      GO TO 91                                                          GAUS101
9     C(1,1)=0.                                                         GAUS102
   91 RETURN                                                            GAUS103
         END                                                            GAUS104-
```

INPUT PARAMETER FORM FOR SUBROUTINE CHAPX

An ionospheric electron density model consisting of a Chapman layer with tilts, ripples, and gradients

$$f_N^2 = f_c^2 \exp\left(\alpha\left(1 - z - e^{-z}\right)\right)$$

$$z = \frac{h - h_{max}}{H}$$

$$f_c^2 = f_{co}^2 \left(1 + A \sin\left(2\pi\left(\theta - \frac{\pi}{2}\right)/B\right) + C\left(\theta - \frac{\pi}{2}\right)\right)$$

$$h_{max} = h_{max_o} + E\left(\theta - \frac{\pi}{2}\right) R_o$$

f_N is the plasma frequency

h is the height above the ground

R_o is the radius of the earth in km

and θ is the colatitude in radians.

Specify:

Critical frequency at the equator, f_{c_o} = _____ MHz (W101)

Height of the maximum electron density at the equator, h_{max_o} = ____ km (W102)

Scale height, H = _____ km (W103)

α = _____ (W104, 0.5 for an α Chapman layer, 1.0 for a β Chapman layer)

Amplitude of periodic variation of f_c^2 with latitude, A = _____ (W105)

Period of variation of f_c^2 with latitude, B = _____ rad / deg / km (W106)

Coefficient of linear variation of f_c^2 with latitude, C = _____ rad^{-1} (W107)

Tilt of the layer, E = _____ rad / deg (W108)

```
      SUBROUTINE CHAPX                                                  CHAP001
C          CHAPMAN LAYER WITH TILTS, RIPPLES, AND GRADIENTS             CHAP002
C          W(104) = 0.5 FOR AN ALPHA-CHAPMAN LAYER                      CHAP003
C                 = 1.0 FOR A BETA-CHAPMAN LAYER                        CHAP004
      COMMON /CONST/ PI,PIT2,PID2,DUM(5)                                CHAP005
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  CHAP006
      COMMON R(6) /WW/ ID(10),WO,W(400)                                 CHAP007
      EQUIVALENCE (THETA,R(2))                                          CHAP008
      EQUIVALENCE (EARTHR,W(2)),(F,W(6)),(FC,W(101)),(HM,W(102)),       CHAP009
     1 (SH,W(103)),(ALPHA,W(104)),(A,W(105)),(B,W(106)),(C,W(107)),     CHAP010
     2 (E,W(108)),(PERT,W(150))                                         CHAP011
      DATA (MODX(1)=6H CHAPX)                                           CHAP012
      ENTRY ELECTX                                                      CHAP013
      THETA2=THETA-PID2                                                 CHAP014
      HMAX=HM+EARTHR*E*THETA2                                           CHAP015
      H=R(1)-EARTHR                                                     CHAP016
      Z=(H-HMAX)/SH                                                     CHAP017
      D=0.                                                              CHAP018
      IF (B.NE.0.) D=PIT2/B                                             CHAP019
      TEMP=1.+A*SIN(D*THETA2)+C*THETA2                                  CHAP020
      EXZ=1.-EXP(-Z)                                                    CHAP021
      X=(FC/F)**2*TEMP*EXP(ALPHA*(EXZ-Z))                               CHAP022
      PXPR=-ALPHA*X*EXZ/SH                                              CHAP023
      PXPTH=X*(D*A*SIN(PID2-D*THETA2)+C)/TEMP-PXPR*EARTHR*E             CHAP024
      PXPPH=0.                                                          CHAP0245
      IF (PERT.NE.0.) CALL ELECT1                                       CHAP025
      RETURN                                                            CHAP026
         END                                                            CHAP 27-
```

INPUT PARAMETER FORM FOR SUBROUTINE VCHAPX

An ionospheric electron density model consisting of a Chapman layer with variable scale height

$$f_N^2 = f_c^2 \, \tau^{\frac{1}{2}} \, e^{\frac{1}{2}(1 - \tau)}$$

$$\tau = \left(\frac{h_{max}}{h}\right)^{\chi}$$

h is the height above the ground.

Specify:

critical frequency, fc = _____ MHz (W101)

height of maximum electron density, h_{max} = _____ km (W102)

χ = _____ (W103)

```
      SUBROUTINE VCHAPX                                                 VCHA001
C          CHAPMAN LAYER WITH VARIABLE SCALE HEIGHT                     VCHA002
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  VCHA003
      COMMON R(6) /WW/ ID(10),W0,W(400)                                 VCHA004
      EQUIVALENCE (EARTHR,W(2)),(F,W(6)),(FC,W(101)),(HM,W(102)),       VCHA005
     1 (CHI,W(103)),(PERT,W(150))                                       VCHA006
      DATA (MODX(1)=6HVCHAPX)                                           VCHA007
      ENTRY ELECTX                                                      VCHA008
      HMAX=HM                                                           VCHA009
      X=PXPR=PXPTH=PXPPH=0.                                             VCHA010
      H=R(1)-EARTHR                                                     VCHA011
      IF (H.LE.0.) GO TO 50                                             VCHA012
      TAU=(HM/H)**CHI                                                   VCHA013
      X=(FC/F)**2*SQRT(TAU)*EXP(0.5*(1.-TAU))                           VCHA014
      PXPR=.5*X*(TAU-1.)*CHI/H                                          VCHA015
   50 IF (PERT.NE.0.) CALL ELECT1                                       VCHA016
      RETURN                                                            VCHA017
         END                                                            VCHA018-
```

INPUT PARAMETER FORM FOR SUBROUTINE DCHAPT

An ionospheric electron density model consisting of a double, tilted α-Chapman layer

$$f_N^2 = f_{c1}^2 \exp \frac{1}{2} (1 - z_1 - e^{-z_1}) + f_{c2}^2 \exp \frac{1}{2} (1 - z_2 - e^{-z_2})$$

$$z_1 = \frac{h - h_{m1}}{H_1} \quad ; \quad z_2 = \frac{h - h_{m2}}{H_2}$$

$$f_{c1}^2 = f_{c10}^2 \; C(\theta - \pi/2)$$

$$f_{c2}^2 = f_{c20}^2 \; C(\theta - \pi/2)$$

$$h_{m1} = h_{m10} + R_o E \left(\frac{\pi}{180}\right) \left(\theta - \frac{\pi}{2}\right)$$

$$h_{m2} = h_{m20} + R_o E \left(\frac{\pi}{180}\right) \left(\theta - \frac{\pi}{2}\right)$$

Specify:

f_{c10}	= _____	MHz (f_{c1} at equator)	(W101)
h_{m10}	= _____	Km (h_{m1} at equator)	(W102)
H_1	= _____	Km	(W103)
f_{c20}	= _____	MHz (f_{c2} at equator)	(W104)
h_{m20}	= _____	Km (h_{m2} at equator)	(W105)
H_2	= _____	Km	(W106)
C	= _____	rad^{-1} (fractional change in f_{c1}, f_{c2}, position for increases southward)	(W107)
E	= _____	deg (positive for upward tilt to the south)	(W108)

```
      SUBROUTINE DCHAPT                                                 DCHA001
C          TWO CHAPMAN LAYERS WITH TILTS                                DCHA002
      COMMON /CONST/ PI,PIT2,PID2,DUM(5)                                DCHA003
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  DCHA004
      COMMON R(6) /WW/ ID(10),W0,W(400)                                 DCHA005
      EQUIVALENCE (EARTHR,W(2)),(F,W(6)),(FC1,W(101)),(HM1,W(102)),     DCHA006
     1 (SH1,W(103)),(FC2,W(104)),(HM2,W(105)),(SH2,W(106)),(C,W(107)),  DCHA007
     2 (E,W(108)),(PERT,W(150))                                         DCHA008
      DATA (MODX(1)=6HDCHAPT)                                           DCHA009
      ENTRY ELECTX                                                      DCHA010
      EARTHE=EARTHR*E                                                   DCHA011
      THETA2=R(2)-PID2                                                  DCHA012
      HMAX=HM1+EARTHE*THETA2                                            DCHA013
      X=PXPR=PXPTH=PXPPH=0.                                             DCHA014
      H=R(1)-EARTHR                                                     DCHA015
      IF (H.LT.0.) GO TO 50                                             DCHA016
      Z1=(H-HMAX)/SH1                                                   DCHA017
      EXPZ1=1.-EXP(-Z1)                                                 DCHA018
      TEMP=1.+C*THETA2                                                  DCHA019
      X=(FC1/F)**2*TEMP*EXP(.5*(EXPZ1-Z1))                              DCHA020
      PXPR=-0.5*X*EXPZ1/SH1                                             DCHA021
      PXPTH=X*C/TEMP-PXPR*EARTHE                                        DCHA022
      IF (FC2.EQ.0.) GO TO 50                                           DCHA023
      Z2=(H-HM2-EARTHE*THETA2)/SH2                                      DCHA024
      EXPZ2=1.-EXP(-Z2)                                                 DCHA025
      X2=(FC2/F)**2*TEMP*EXP(.5*(EXPZ2-Z2))                             DCHA026
      X=X+X2                                                            DCHA027
      PXPR2=-0.5*X2*EXPZ2/SH2                                           DCHA028
      PXPR=PXPR+PXPR2                                                   DCHA029
      PXPTH=PXPTH+X2*C/TEMP-PXPR2*EARTHE                                DCHA030
   50 IF (PERT.NE.0.) CALL ELECT1                                       DCHA031
      RETURN                                                            DCHA032
         END                                                            DCHA033-
```

INPUT PARAMETER FORM FOR SUBROUTINE LINEAR

An ionospheric electron density model consisting of a linear layer

$N = 0$ for $h \leq h_{min}$

$N = A(h - h_{min})$ for $h > h_{min}$

The ray will penetrate if $h > h_{max}$.

Specify:

A = _____ electrons/cm^3/ km (W101)

h_{max} = _____ km (W102)

h_{min} = _____ km (W103)

```
      SUBROUTINE LINEAR                                                 LINE001
C        LINEAR ELECTRON DENSITY MODEL                                  LINE002
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RAD,K,DUM(2)                     LINE003
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  LINE004
      COMMON R(6) /WW/ ID(10),W0,W(400)                                 LINE005
      EQUIVALENCE (EARTHR,W(2)),(F,W(6)),(FACT,W(101)),(HM,W(102)),     LINE006
     1 (HMIN,W(103)),(PERT,W(150))                                      LINE007
      REAL K                                                            LINE008
      DATA (MODX(1)=6HLINEAR)                                           LINE009
      ENTRY ELECTX                                                      LINE010
      H=R(1)-EARTHR                                                     LINE011
      HMAX=HM                                                           LINE012
      X=PXPR=PXPTH=PXPPH=0.                                             LINE013
      IF (H.LE.HMIN) GO TO 50                                           LINE014
      PXPR=K*FACT/F**2                                                  LINE015
      X=PXPR*(H-HMIN)                                                   LINE016
   50 IF (PERT.NE.0.) CALL ELECT1                                       LINE017
      RETURN                                                            LINE018
         END                                                            LINE019-
```

INPUT PARAMETER FORM FOR SUBROUTINE QPARAB

An ionospheric electron density model consisting of a parabolic or a quasi-parabolic layer (concentric)

$$f_N^2 = f_c^2 \left[1. - \frac{h-h_{max}}{Y_m} \cdot C^2\right] \qquad \text{if } f_N^2 > 0.$$

$$f_N^2 = 0. \quad \text{otherwise.}$$

$C = 1.$ for a parabolic layer

$$C = \frac{R_o + h_{max} - Y_m}{R_o + h} \quad \text{for a quasi-parabolic layer}$$

where R_o is the radius of the earth.

Specify:

Critical frequency, f_c = _____Mc/s (W101)

Height of maximum electron density, h_{max} = _____km. (W102)

Semi-thickness, Y_m = _____km. (W103)

Type of profile:

Plain parabolic_____(W104 = 0.)

Quasi-parabolic_____(W104 = 1.)

```
      SUBROUTINE QPARAB                                                 PARA001
C        PLAIN PARABOLIC OR QUASI-PARABOLIC PROFILE                     PARA002
C        W(104) = 0. FOR A PLAIN PARABOLIC PROFILE                      PARA003
C               = 1. FOR A QUASI-PARABOLIC PROFILE                      PARA004
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  PARA005
      COMMON R(6) /WW/ ID(10),W0,W(400)                                 PARA006
      EQUIVALENCE (EARTHR,W(2)),(F,W(6)),(FC,W(101)),(HM,W(102)),       PARA007
     1 (YM,W(103)),(QUASI,W(104)),(PERT,W(150))                         PARA008
      DATA (MODX(1)=6HQPARAB)                                           PARA009
      ENTRY ELECTX                                                      PARA010
      HMAX=HM                                                           PARA011
      X=PXPR=PXPTH=PXPPH=0.                                             PARA012
      H=R(1)-EARTHR                                                     PARA013
      FCF2=(FC/F)**2                                                    PARA014
      CONST=1.                                                          PARA015
      IF (QUASI.EQ.1.) CONST=(EARTHR+HM-YM)/R(1)                        PARA016
      Z=(H-HM)/YM*CONST                                                 PARA017
      X=MAX1F(0.,FCF2*(1.-Z*Z))                                         PARA018
      IF (X.EQ.0.) GO TO 50                                             PARA019
      IF (QUASI.EQ.1.) CONST=(EARTHR+HM)*(EARTHR+HM-YM)/R(1)**2         PARA020
      PXPR=-2.*Z*FCF2/YM*CONST                                          PARA021
   50 IF (PERT.NE.0.) CALL ELECT1                                       PARA022
      RETURN                                                            PARA023
         END                                                            PARA024-
```

INPUT PARAMETER FORM FOR SUBROUTINE BULGE

An analytic ionospheric electron density model which represents the general latitude variation of the equatorial ionosphere (afternoon, equinox, sunspot maximum) - see the center panel of figure 3.18b, page 133 of Davies (1965).

The model is an alpha Chapman layer with parameters which vary with geomagnetic latitude.

$$f_N^2 = f_c^2 \, e^{\frac{1}{2}(1 - z - e^{-z})}$$

where $z = \dfrac{h - h_{max}}{H}$

f_N is the plasma frequency

f_c is the critical frequency

h_{max} is the height of the maximum electron density

H is the scale height

h is height

---

f_c, h_{max}, H vary with geomagnetic latitude in the following way:

if $h < 100$ km, $h_{max} = 350$ km, $f_c = 15$ Mc/s

For $h \geq 100$ km,

$h_{max} = 350$ if $\lambda \geq 24°$

$h_{max} = 430 + 80 \cos\left(\frac{180}{24}\lambda\right)$ if $\lambda < 24°$

λ is the geomagnetic latitude in degrees

$$f_c = \sqrt{50\left(\frac{\lambda}{8}\right)^2 \exp\left(2 - \left|\frac{\lambda}{8}\right|\right) + 40}$$

In all cases H is determined by the constraint that

$f_N = 2$ Mc/s at 100 km.

```
      SUBROUTINE BULGE                                                  BULG001
C        ANALYTICAL MODEL OF THE VARIATION OF THE EQUATORIAL F2 LAYER   BULG002
C        IN GEOMAGNETIC LATITUDE  (EQUATORIAL BULGE AND ANOMALY)        BULG003
C        SEE FIGURE 3.18B, PAGE 133 IN DAVIES (1965).                   BULG004
C        THIS MODEL HAS NO VARIATION IN GEOMAGNETIC LONGITUDE.          BULG005
      COMMON /CONST/ PI,PIT2,PID2,DUM(5)                                BULG006
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  BULG007
      COMMON R(6) /WW/ ID(10),W0,W(400)                                 BULG008
      EQUIVALENCE (EARTHR,W(2)),(F,W(6)),(PERT,W(150))                  BULG009
      DATA (MODX(1)=6H BULGE)                                           BULG010
      ENTRY ELECTX                                                      BULG011
      H=R(1)-EARTHR                                                     BULG012
      PHMPTH=PFC2PTH=0.                                                 BULG013
      HMAX=350.                                                         BULG014
      FC2=225.                                                          BULG015
      IF(H.LT.100.) GO TO 2                                             BULG016
C        EQUATORIAL BULGE                                               BULG017
      BULLAT=7.5*(PID2-R(2))                                            BULG018
      IF(ABS(BUL LAT).GE.PI)  GO TO 1                                   BULG019
      HMAX=430.+80.*COS(BULLAT)                                         BULG020
      PHMPTH=600.*SIN(BUL LAT)                                          BULG021
C        EQUATORIAL ANOMALY                                             BULG022
    1 ANMLAT=22.5*(PID2-R(2))/PI                                        BULG023
      POW=2.-ABS(ANM LAT)                                               BULG024
      FC2=50.*ANM LAT**2*EXP(  POW     ) + 40.                          BULG025
      PFC2PTH=-1125./PI*POW*ANMLAT*EXP(POW)                             BULG026
C    FORCING PLASMA FREQ AT 100 KM TO BE 2 MHZ IN ORDER TO CALCULATE SH BULG027
    2 ALPHA=2.*ALOG(FC2/4.)+1.                                          BULG028
      Z100=-ALOG(ALPHA)                                                 BULG029
      DO 3 I=1,5                                                        BULG030
    3 Z100=-ALOG(ALPHA-Z100)                                            BULG031
      SH=(100.-HMAX)/Z100                                               BULG032
      Z=(H-HMAX)/SH                                                     BULG033
      EXZ=1.-EXP(-Z)                                                    BULG034
      X=FC2*EXP(.5*(EXZ-Z))/F**2                                        BULG035
      PXPR=-0.5*X*EXZ/SH                                                BULG036
      PXPTH=-PXPR*(1.-Z/Z100)*PHMPTH+(1.-Z*EXZ/(Z100*(1.-EXP(-Z100))))  BULG037
     1       *X/FC2*PFC2PTH                                             BULG038
      PXPPH=0.                                                          BULG0385
      IF (PERT.NE.0.) CALL ELECT1                                       BULG039
      RETURN                                                            BULG040
         END                                                            BULG041-
```

INPUT PARAMETER FORM FOR SUBROUTINE EXPX

An exponential electron density profile

$$N = N_o e^{a(h-h_o)}$$

h is the height above the ground.

Specify:

the electron density at the height h_o, N_o = _____ cm^{-3} (W101)

the reference height, h_o = _____ km (W102)

the exponential increase of N with height, a = _____ km^{-1} (W103)

```
      SUBROUTINE EXPX                                                   EXPX001
C        EXPONENTIAL ELECTRON DENSITY MODEL                             EXPX002
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RAD,K,DUM(2)                     EXPX003
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  EXPX004
      COMMON R(6) /WW/ ID(10),W0,W(400)                                 EXPX005
      EQUIVALENCE (EARTHR,W(2)),(F,W(6)),                               EXPX006
     1 (N0,W(101)),(H0,W(102)),(A,W(103)),(PERT,W(150))                 EXPX007
      REAL N, N0 ,K                                                     EXPX008
      DATA (MODX(1)=4HEXPX),(HMAX=350.)                                 EXPX009
      ENTRY ELECTX                                                      EXPX010
      H=R(1)-EARTHR                                                     EXPX011
      N =N0 * EXP(A*(H-H0))                                             EXPX012
      X=K*N/F**2                                                        EXPX013
      PXPR=A*X                                                          EXPX014
      PXPTH=PXPPH=0.                                                    EXPX0145
      IF (PERT.NE.0.) CALL ELECT1                                       EXPX015
      RETURN                                                            EXPX016
         END                                                            EXPX017-
```

APPENDIX 4. PERTURBATIONS TO ELECTRON DENSITY MODELS WITH INPUT PARAMETER FORMS

The following perturbations to electron density models (irregularities) are available. The input parameter forms, which describe the perturbation, and the subroutine listings are given on the pages shown.

a.	Do-nothing perturbation (ELECT1)	126
b.	East-west irregularity with an elliptical cross-section above the equator (TORUS)	127
c.	Two east-west irregularities with elliptical cross-sections above the equator (DTORUS)	129
d.	Increase in electron density at any latitude (TROUGH)	131
e.	Increase in electron density produced by a shock wave (SHOCK)	132
f.	"Gravity-wave" irregularity (WAVE)	134
g.	"Gravity-wave" irregularity (WAVE2)	136
h.	Height profile of time derivative of electron density for calculating Doppler shift (DOPPLER)	138

To add other perturbations to electron density models the user must write a subroutine to modify the normalized electron density (X) and its gradient ($\partial X/\partial r$, $\partial X/\partial \theta$, $\partial X/\partial \varphi$) as a function of position in spherical polar coordinates (r, θ, φ).

The restrictions on electron density models also apply to perturbations. Again, the coordinates r, θ, φ refer to the computational coordinate system, which may not be the same as geographic coordinates. In particular, they are geomagnetic coordinates when the earth-centered dipole model of the earth's magnetic field is used.

The input to the subroutine is through blank common (see Table 3) for the position (r, θ, φ) and through common block /XX/ (see Table 8) for the unperturbed electron density and its gradient. The output is through common block /XX/. It is useful if the name of the subroutine suggests the perturbation model to which it corresponds. It should have an entry point ELECT1 so that it may be called by an electron density subroutine. Any parameters needed by the subroutine should be input into W151 through W199 of the W array. (See Table 2.)

If no perturbation is wanted, the following subroutine should be used.

```
      SUBROUTINE ELECT1                                                 ELEC001
C          USE WHEN AN ELECTRON DENSITY PERTURBATION IS NOT WANTED      ELEC002
      COMMON /XX/ MODX(2),X(6)                                          ELEC003
      COMMON /WW/ ID(10),W0,W(400)                                      ELEC004
      EQUIVALENCE (PERT,W(150))                                         ELEC005
      DATA (MODX(2)=6H NONE )                                           ELEC006
      PERT=0.                                                           ELEC007
      RETURN                                                            ELEC008
         END                                                            ELEC009-
```

INPUT PARAMETER FORM FOR SUBROUTINE TORUS

A perturbation to an ionospheric electron density model consisting of an East-West irregularity with an elliptical cross section above the equator

$$N = N_0 (1 + \Delta)$$

$$\Delta = C_0 \exp \left\{ - \left[\frac{(R_0 + H_0)(\theta - \pi/2) \cos \beta + (R - R_0 - H_0) \sin \beta}{A} \right]^2 - \left[\frac{(R - R_0 - H_0) \cos \beta - (R_0 + H_0)(\theta - \pi/2) \sin \beta}{B} \right]^2 \right\}$$

R_0 is the radius of the earth.

R, θ, φ give the position in spherical polar coordinates.

$N_0(R, \theta, \varphi)$ is any ionospheric electron density model.

Specify:

C_0 = _____. (W151)

Semi-major axis of ellipse, A = _____km (W152)

Semi-minor axis of ellipse, B = _____km (W153)

Tilt of ellipse, β = _____ degrees (W154)

Height of torus from ground, H_0 = _____ km (W155)

(W150: = 1. to use perturbation, = 0. to ignore perturbation)

```
      SUBROUTINE TORUS                                                  TOR 001
      COMMON /CONST/ PI,PIT2,PID2,DUM(5)                                TOR 002
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  TOR 003
      COMMON R(6) /WW/ ID(10),W0,W(400)                                 TOR 004
      EQUIVALENCE (EARTHR,W(2)),(C0,W(151)),(A,W(152)),(B,W(153)),      TOR 005
     1 (BETA,W(154)),(H0,W(155))                                        TOR 006
      REAL LAMBDA                                                       TOR 007
      DATA (PDPP=0.),(MODX(2)=6H TORUS)                                 TOR 008
      ENTRY ELECT1                                                      TOR 009
      IF (X.EQ.0..AND.PXPR.EQ.0..AND.PXPTH.EQ.0..AND.PXPPH.EQ.0.) RETURNTOR 010
      IF (C0.EQ.0.) RETURN                                              TOR 011
      R0=EARTHR+H0                                                      TOR 012
      Z=R(1)-R0                                                         TOR 013
      LAMBDA=R0*(R(2)-PID2)                                             TOR 014
      SINBET=SIN(BETA)                                                  TOR 015
      COSBET=COS(BETA)                                                  TOR 016
      P=LAMBDA*COSBET+Z*SINBET                                          TOR 017
      Y=Z*COSBET-LAMBDA*SINBET                                          TOR 018
      DELTA=C0*EXP(-(P/A)**2-(Y/B)**2)                                  TOR 019
      DEL1=DELTA+1.                                                     TOR 020
      PDPR=-2.*DELTA*(P*SINBET/A**2+Y*COSBET/B**2)                      TOR 021
      PDPT=-2.*DELTA*(P*R0*COSBET/A**2-Y*R0*SINBET/B**2)                TOR 022
      PXPR=PXPR*DEL1+X*PDPR                                             TOR 023
      PXPTH=PXPTH*DEL1+X*PDPT                                           TOR 024
      PXPPH=PXPPH*DEL1+X*PDPP                                           TOR 025
      X=X*DEL1                                                          TOR 026
      RETURN                                                            TOR 027
         END                                                            TOR 028-
```

INPUT PARAMETER FORM FOR SUBROUTINE DTORUS

A perturbation to an ionospheric electron density model consisting of two east-west irregularities with elliptical cross sections above the equator. Since the model is expressed in spherical coordinates and does not depend on longitude, the perturbation is actually a torus circling the earth above the equator.

$$N = N_0 \, (1 + \Delta)$$

$$\Delta = C_1 \exp\left\{-\left[\frac{(r_0 + H_1)(\theta - \pi/2)\cos\beta + (r - r_0 - H_1)\sin\beta}{A_1}\right]^2 - \left[\frac{(r - r_0 - H_1)\cos\beta - (r_0 + H_1)(\theta - \pi/2)\sin\beta}{B_1}\right]^2\right\}$$

$$+ C_2 \exp\left\{-\left[\frac{(r_0 + H_2)(\theta - \pi/2 + \delta\theta)\cos\beta + (r - r_0 - H_2)\sin\beta}{A_2}\right] - \left[\frac{(r - r_0 - H_2)\cos\beta - (r_0 + H_2)(\theta - \pi/2 + \delta\theta)\sin\beta}{B_2}\right]^2\right\}.$$

$\delta\theta$ = Northward angular displacement of the lower blob from the upper one

$$= \frac{H_1 - H_2}{\tan\beta\,(r_0 + H_2)} \quad .$$

r_0 is the radius of the earth.

r, θ, φ are spherical (earth-centered) polar coordinates.

$N_0(r, \theta, \varphi)$ is any electron density model.

Specify:

use perturbation_____(W150 = 1.)
ignore perturbation_____(W150 = 0.)

Fractional perturbation electron density at the center of the upper blob, C_1 = _____(W151).

That of the lower blob, C_2 = _____(W156).

Height (above ground) of the center of the upper blob, H_1 = _____km (W155).

That of the lower blob, H_2 = _____ km (W159).

Angle (with a horizontal southward vector) of the line joining the blob centers, θ = _____ rad / deg (W154).

Semi-axis of the upper blob, to the 1/e perturbation contour, in the direction of the line joining the blobs, A_1 = _____ km (W152).

That of the lower blob, A_2 = _____ km (W157).

Semi-axis of the upper blob in the direction normal to the line joining the blobs, B_1 = _____ km (W153).

That of the lower blob, B_2 = _____ km (W158).

```
      SUBROUTINE DTORUS                                                 DTOR001
      COMMON /CONST/ PI,PIT2,PID2,DUM(5)                                DTOR002
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT                       DTOR003
      COMMON R(6) /WW/ ID(10),W0,W(400)                                 DTOR004
      EQUIVALENCE (EARTHR,W(2)),(C1,W(151)),(A1,W(152)),(B1,W(153)),    DTOR005
     1 (BETA,W(154)),(H1,W(155)),(C2,W(156)),(A2,W(157)),(B2,W(158)),   DTOR006
     2 (H2,W(159))                                                      DTOR007
      REAL LAMBDA1,LAMBDA2                                              DTOR008
      DATA (MODX(2)=6HDTORUS),(PDPP=0.)                                 DTOR009
      ENTRY ELECT1                                                      DTOR010
      IF (X.EQ.0..AND.PXPR.EQ.0..AND.PXPTH.EQ.0..AND.PXPPH.EQ.0.) RETURNDTOR011
      IF (C1.EQ.0.) RETURN                                              DTOR012
      R1=EARTHR+H1                                                      DTOR013
      R2=EARTHR+H2                                                      DTOR014
      Z1=R(1)-R1                                                        DTOR015
      Z2=R(1)-R2                                                        DTOR016
      LAMBDA1=R1*(R(2)-PID2)                                            DTOR017
      LAMBDA2=R2*(R(2)-PID2+(H1-H2)/R2/TANF(BETA))                      DTOR018
      SINBET=SIN(BETA)                                                  DTOR019
      COSBET=COS(BETA)                                                  DTOR020
      P1=LAMBDA1*COSBET+Z1*SINBET                                       DTOR021
      P2=LAMBDA2*COSBET+Z2*SINBET                                       DTOR022
      Y1=Z1*COSBET-LAMBDA1*SINBET                                       DTOR023
      Y2=Z2*COSBET-LAMBDA2*SINBET                                       DTOR024
      DELTA1=C1*EXP(-(P1/A1)**2-(Y1/B1)**2)                             DTOR025
      DELTA2=C2*EXP(-(P2/A2)**2-(Y2/B2)**2)                             DTOR026
      DEL1=1.+DELTA1+DELTA2                                             DTOR027
      PDPR1=-2.*DELTA1*(P1*SINBET/A1**2+Y1*COSBET/B1**2)                DTOR028
      PDPR2=-2.*DELTA2*(P2*SINBET/A2**2+Y2*COSBET/B2**2)                DTOR029
      PDPT1=-2.*DELTA1*(P1*R1*COSBET/A1**2-Y1*R1*SINBET/B1**2)          DTOR030
      PDPT2=-2.*DELTA2*(P2*R2*COSBET/A2**2-Y2*R2*SINBET/B2**2)          DTOR031
      PXPR=PXPR*DEL1+X*(PDPR1+PDPR2)                                    DTOR032
      PXPTH=PXPTH*DEL1+X*(PDPT1+PDPT2)                                  DTOR033
      PXPPH=PXPPH*DEL1*PDPP                                             DTOR034
      X=X*DEL1                                                          DTOR035
      RETURN                                                            DTOR036
         END                                                            DTOR037-
```

INPUT PARAMETER FORM FOR SUBROUTINE TROUGH

A perturbation to an ionospheric electron density model consisting of an increase in electron density near any latitude

$$N = (1 + \Delta)\, N_0\, (R, \theta, \varphi) \qquad W = B \text{ for } \frac{\pi}{2} - \theta - \lambda \geq 0$$

$$\Delta = A \exp\left(-\left(\frac{\pi/2 - \theta - \lambda}{W}\right)^2\right) \qquad W = B \times C \text{ for } \frac{\pi}{2} - \theta - \lambda < 0$$

$N_0\,(R, \theta, \varphi)$ is any ionospheric electron density model.

R, θ, φ give the position in spherical polar coordinates.

Specify:

Amplitude of the perturbation, A = _____ (W151)

half width of the perturbation, B = _____ degrees (W152)

latitude of the perturbation, λ = _____ degrees (W153)

width factor for South of trough, C = _____ (W154)

(W150: = 1. to use perturbation, = 0. to ignore perturbation)

```
      SUBROUTINE TROUGH                                                 TROU001
C            A PERTURBATION TO AN ELECTRON DENSITY MODEL                TROU002
      COMMON /CONST/ PI,PIT2,PID2,DUM(5)                                TROU003
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  TROU004
      COMMON R(6) /WW/ ID(10),W0,W(400)                                 TROU005
      EQUIVALENCE (A,W(151)),(B,W(152)),(ALAT,W(153)),(FACTOR,W(154))   TROU006
      DATA (MODX(2)=6HTROUGH)                                           TROU007
      ENTRY ELECT1                                                      TROU008
      IF (X.EQ.0..AND.PXPR.EQ.0..AND.PXPTH.EQ.0..AND.PXPPH.EQ.0.) RETURNTROU009
      IF (A.EQ.0.) RETURN                                               TROU010
      ANGLE=R(2)+ALAT-PID2                                              TROU011
      WIDTH=B                                                           TROU012
      IF (ANGLE.GT.0.)   WIDTH=FACTOR*B                                 TROU013
      ANGLE=ANGLE/WIDTH                                                 TROU014
      DELTA=A*EXP(-ANGLE**2)                                            TROU015
      DEL1=DELTA+1.                                                     TROU016
      PXPR=PXPR*DEL1                                                    TROU017
      PXPTH=PXPTH*DEL1-2.*X*ANGLE*DELTA/WIDTH                           TROU018
      PXPPH=PXPPH*DEL1                                                  TROU019
      X=X*DEL1                                                          TROU020
      RETURN                                                            TROU021
         END                                                            TROU022-
```

INPUT PARAMETER FORM FOR SUBROUTINE SHOCK

A perturbation to an ionospheric electron density model consisting of an increase in electron density produced by a shock wave

$$N(R, \theta, \varphi) = N_0(R, \theta, \varphi)[1 + P \exp(-9\left(\frac{\rho_c - \rho}{w}\right)^2)]$$

$$\rho_c = s(h - h_0) - w$$

$$\rho = R\left|\cos^{-1}[\cos(\varphi - \varphi_0)\cos(\lambda - \lambda_0)]\right|$$

$N_0(R, \theta, \varphi)$ is the ambient electron density specified by any electron density model.

R, θ, φ give the position in spherical polar coordinates.

$h = R - a$ is the height above the surface of the earth.

a is the radius of the earth.

$\lambda = \frac{\pi}{2} - \theta$ is the latitude.

Specify:

Relative increase in electron density, P = _____ (W151).

Width of the disturbance, w = _____ km (W152).

Latitude of the center of the disturbance, λ_0 = _____ radians or degrees (W153).

Longitude of the center of the disturbance, φ_0 = _____ radians or degrees (W154).

Slope measured from vertical - rate of increase of ρ_c with height, s = _____ (W155).

Height to the bottom of the disturbance, h_0 = _____ km (W156).

(W150: = 1. to use perturbation, = 0. to ignore perturbation)

```
      SUBROUTINE SHOCK                                                  SHOC001
C   A PERTURBATION TO AN ELECTRON DENSITY MODEL SIMULATING A SHOCK WAVE SHOC002
      COMMON /CONST/ PI,PIT2,PID2,DUM(5)                                SHOC003
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  SHOC004
      COMMON R(6) /WW/ ID(10),WO,W(400)                                 SHOC005
      EQUIVALENCE (EARTHR,W(2)),(P,W(151)),(WW,W(152)),(ALAT,W(153)),   SHOC006
     1 (ALON,W(154)),(S,W(155)),(HO,W(156))                             SHOC007
      REAL LAT,LON                                                      SHOC008
      DATA (MODX(2)=6H SHOCK)                                           SHOC009
      ENTRY ELECT1                                                      SHOC010
      IF (X.EQ.0..AND.PXPR.EQ.0..AND.PXPTH.EQ.0..AND.PXPPH.EQ.0.) RETURNSHOC011
      IF (P.EQ.0..OR.WW.EQ.0.)  RETURN                                  SHOC012
      H=R(1)-EARTHR                                                     SHOC013
      RHOC=S*(H-HO)-WW                                                  SHOC014
      LON=R(3)-ALON                                                     SHOC015
      LAT=PID2-R(2)-ALAT                                                SHOC016
      COSLON=COS(LON)                                                   SHOC017
      COSLAT=COS(LAT)                                                   SHOC018
      U=COSLON*COSLAT                                                   SHOC019
      RHO=R(1)*ACOS(U)                                                  SHOC020
      DIF=RHOC-RHO                                                      SHOC021
      CON=-9./WW**2                                                     SHOC022
      CONS=P*EXP(CON*DIF**2)                                            SHOC023
      CONST=1.+CONS                                                     SHOC024
      CON=2.*CON*CONS*DIF                                               SHOC025
      PXPR=PXPR*CONST+X*CON*(S-RHO/R(1))                                SHOC026
      CONS=R(1)*(1./SQRT(1.-U**2))                                      SHOC027
      PXPTH=PXPTH*CONST+X*CON*CONS*COSLON*SIN(LAT)                      SHOC028
      PXPPH=PXPPH*CONST-X*CON*CONS*COSLAT*SIN(LON)                      SHOC029
      X=X*CONST                                                         SHOC030
      RETURN                                                            SHOC031
         END                                                            SHOC032-
```

INPUT PARAMETER FORM FOR SUBROUTINE WAVE

A perturbation to an ionospheric electron density model consisting of a "gravity-wave" irregularity traveling from north pole to south pole

$$N = N_0(1 + \Delta)$$

$$\Delta = \delta \exp\left\{- [(R - R_0 - z_0)/H]^2\right\} \cdot$$

$$\cos\left\{2\pi \left[t' + (\pi/2 - \theta)\frac{R_0}{\lambda_x} + (R - R_0)/\lambda_z\right]\right\}$$

$$\frac{\partial N}{\partial t} = \frac{-2\pi}{\lambda_x} V_x N_0 \delta \exp - [(R - R_0 - z_0)/H]^2 \cdot$$

$$\sin 2\pi \left[t' + (\pi/2 - \theta)\frac{R_0}{\lambda_x} + (R - R_0)/\lambda z\right]$$

R_0 is the radius of the earth.

R, θ, φ are the spherical (earth-centered) polar coordinates (Δ is independent of φ).

N_0 (R, θ, φ) is any electron density model.

Specify:

the height of maximum wave amplitude, z_0 = _____km (W151)

wave-amplitude "scale height," H = _____km (W152)

wave perturbation amplitude, δ = _____ [0. to 1.] (W153)

horizontal trace velocity, V_x = _____km/sec (W154)
(needed only if Doppler shift is calculated)

horizontal wavelength, λ_x = _____km (W155)

vertical wavelength, λ_z = _____km (W156)

time in wave periods, t' = _____ [0. to 1.] (W157)

(W150: = 1. to use perturbation, = 0. to ignore perturbation)

```
      SUBROUTINE WAVE                                                   WAVE001
C          PERTURBATION TO AN ALPHA-CHAPMAN ELECTRON DENSITY MODEL      WAVE002
      COMMON /CONST/ PI,PIT2,PID2,DUM(5)                                WAVE003
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  WAVE004
      COMMON R(6) /WW/ ID(10),WO,W(400)                                 WAVE005
      EQUIVALENCE (EARTHR,W(2)),(ZO,W(151)),(SH,W(152)),(DELTA,W(153)), WAVE006
     1 (VSUBX,W(154)),(LAMBDAX,W(155)),(LAMBDAZ,W(156)),(TP,W(157))     WAVE007
      REAL LAMBDAX,LAMBDAZ                                              WAVE008
      DATA (MODX(2)=6H WAVE )                                           WAVE009
      ENTRY ELECT1                                                      WAVE010
      IF (X.EQ.0..AND.PXPR.EQ.0..AND.PXPTH.EQ.0..AND.PXPPH.EQ.0.) RETURNWAVE011
      IF (DELTA.EQ.0..OR.SH.EQ.0.)    RETURN                            WAVE012
      H=R(1)-EARTHR                                                     WAVE013
      EXPQ=EXP(-((H-ZO)/SH)**2)                                         WAVE014
      TMP=PIT2*(TP+(PID2-R(2))*EARTHR/LAMBDAX+H/LAMBDAZ)                WAVE015
      SINW=SIN(TMP)                                                     WAVE016
      COSW=SIN(PID2-TMP)                                                WAVE017
      CONS=1.0+DELTA*EXPQ*COSW                                          WAVE018
      IF (H.NE.0.) PXPR=PXPR*CONS-X*DELTA*EXPQ*(2.0/SH**2*(H-ZO)*COSW   WAVE019
     1 +PIT2/LAMBDAZ*SINW)                                              WAVE020
      PXPTH=PXPTH*CONS+X*DELTA*PIT2*EARTHR/LAMBDAX*SINW*EXPQ            WAVE021
      PXPPH=PXPPH*CONS                                                  WAVE022
      PXPT=0.                                                           WAVE023
      IF (VSUBX.NE.0.) PXPT=-PIT2*VSUBX/LAMBDAX*X*DELTA*EXPQ*SINW       WAVE024
      X=X*CONS                                                          WAVE025
      RETURN                                                            WAVE026
         END                                                            WAVE027-
```

INPUT PARAMETER FORM FOR SUBROUTINE WAVE 2

PERTURBATION TO AN IONOSPHERIC ELECTRON DENSITY MODEL

A "gravity-wave" irregularity traveling from north pole to south pole - same as WAVE 1, but with Gaussian amplitude variations in latitude and longitude, and provision for a horizontal "group velocity "

$$N = N_0 \, (1 + AC)$$

$$A = \delta \exp - \left(\frac{r - r_\oplus - z_\oplus}{H} \right)^2 \cdot \exp - \left(\frac{\theta - \theta_\oplus(t)}{\Theta} \right)^2 \cdot \exp - \left(\frac{\varphi}{\Phi} \right)^2$$

$$C = \cos 2\pi \left[t' + (\pi/2 - \theta) \frac{r_\oplus}{\lambda_x} + (r - r_\oplus)/\lambda_z \right]$$

$$\theta_\oplus = \theta_\infty + V_g \, t / r_\oplus$$

$r_\oplus$ is the radius of the earth.

r, θ, φ are spherical (earth-centered) polar coordinates.

$N_0(r, \theta, \varphi)$ is any electron density model.

Specify:

use perturbation_____(W150 = 1.)

ignore perturbation_____(W150 = 0.)

the height of maximum wave amplitude, $z_\oplus$ = _____km (W151)

wave-amplitude "scale height," H = _____km (W152)

wave perturbation amplitude, δ = _____(0 to 1) (W153)

horizontal trace velocity, V_x = _____km/sec. (W154)
(needed only if Doppler shift is calculated)

horizontal wavelength, λ_x = _____km (W155)

vertical wavelength, λ_z = _____km (W156)

time in wave periods, t' = _____ (W157)

amplitude "scale distance" in latitude, Θ = _____degrees (W159)

amplitude "scale distance" in longitude, Φ = _____degrees (W160)

latitude of maximum amplitude at t = 0, θ_∞ = _____degrees (W158)

southward group velocity, V_g = _____km/sec (W161)
(needed even if Doppler shift is not calculated)

```
      SUBROUTINE WAVE2                                                  WAV2001
C          PERTURBATION TO AN ANY ELECTRON DENSITY MODEL                WAV2002
      COMMON /CONST/ PI,PIT2,PID2,DUM(5)                                WAV2003
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT                       WAV2004
      COMMON R(6) /WW/ ID(10),W0,W(400)                                 WAV2005
      EQUIVALENCE (EARTHR,W(2)),(Z0,W(151)),(SH,W(152)),(DELTA,W(153)), WAV2006
     1 (VSUBX,W(154)),(LAMBDAX,W(155)),(LAMBDAZ,W(156)),(TP,W(157)),    WAV2007
     2 (TH00,W(158)),(THC,W(159)),(PHIC,W(160)),(VGX,W(161))            WAV2008
      REAL LAMBDAX,LAMBDAZ                                              WAV2009
      DATA (MODX(2)=6H WAVE2)                                           WAV2010
      ENTRY ELECT1                                                      WAV2011
      IF (X.EQ.0..AND.PXPR.EQ.0..AND.PXPTH.EQ.0..AND.PXPPH.EQ.0.) RETURNWAV2012
      IF (DELTA.EQ.0..OR.SH.EQ.0.)    RETURN                            WAV2013
      H=R(1)-EARTHR                                                     WAV2014
      TH0=TH00+LAMBDAX*TP*VGX/VSUBX/EARTHR                              WAV2015
      EXPR=EXP(-((H-Z0)/SH)**2)                                         WAV2016
      EXPTH=EXP(-((R(2)-TH0)/THC)**2)                                   WAV2017
      EXPPHI=EXP(-(R(3)/PHIC)**2)                                       WAV2018
      WW=PIT2*(TP+(PID2-R(2))*EARTHR/LAMBDAX+H/LAMBDAZ)                 WAV2019
      SINW=SIN(WW)                                                      WAV2020
      COSW=COS(WW)                                                      WAV2021
      E=DELTA*EXPR*EXPTH*EXPPHI                                         WAV2022
      CONS=1.0+E*COSW                                                   WAV2023
      PXPR=PXPR*CONS-X*E*2.*(COSW*(H-Z0)/SH**2+PI/LAMBDAZ*SINW)         WAV2024
      PXPTH=PXPTH*CONS+2.*E*(X*PI*EARTHR*SINW/LAMBDAX-(R(2)-TH0)/       WAV2025
     1 THC**2*COSW)                                                     WAV2026
      PXPPH=PXPPH*CONS-X*2.*E*R(3)/PHIC**2*COSW                         WAV2027
      PXPT=-PIT2*VSUBX*E/LAMBDAX*SINW+2.0*E*VGX/EARTHR*COSW*(R(2)-TH0   WAV2028
     1 -LAMBDAX*TP/EARTHR)/THC                                          WAV2029
      X=X*CONS                                                          WAV2030
      RETURN                                                            WAV2031
         END                                                            WAV2032-
```

INPUT PARAMETER FORM FOR SUBROUTINE DOPPLER

HEIGHT PROFILE OF dN/dt

(A perturbation to an ionospheric electron density model which calculates the time derivative of electron density for calculating Doppler shifts)

First card tells how many profile points in I4 format. The cards following the first card give the height and dN/dt of the profile points one point per card in F8.2, E12.4 format. The heights must be in increasing order. Set W151 =1.0 to read in a new profile. After the cards are read, DOPPLER will reset W151=0. This subroutine makes an exponential extrapolation down using the bottom 2 points in the profile.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20		1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20	
HEIGHT h km	TIME DERIVATIVE OF ELECTRON DENSITY dN/dt ELECTRONS/cm³/sec	HEIGHT h km	TIME DERIVATIVE OF ELECTRON DENSITY dN/dt ELECTRONS/cm³/sec

```
      SUBROUTINE DOPPLER                                                DOPP001
C      COMPUTES DN/DT                  FROM PROFILES HAVING THE SAME FORMDOPP002
C      AS THOSE USED BY SUBROUTINE TABLE X                              DOPP003
C MAKES AN EXPONENTIAL EXTRAPOLATION DOWN USING THE BOTTOM TWO POINTS   DOPP004
C        NEEDS SUBROUTINE GAUSEL                                        DOPP005
      DIMENSION HPC(250),FN2C(250),ALPHA(250),BETA(250),GAMMA(250),     DOPP006
     1 DELTA(250),SLOPE(250),MAT(4,5)                                   DOPP007
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RAD,K,DUM(2)                     DOPP008
      COMMON /XX/ MODX(2),XDUM,PXPR,PXPTH,PXPPH,X,HMAX                  DOPP009
      COMMON R(6) /WW/ ID(10),WO,W(400)                                 DOPP010
      EQUIVALENCE (EARTHR,W(2)),(F,W(6)),(READFN,W(151))                DOPP011
      REAL MAT,K                                                        DOPP012
      DATA (MODX(2)=7HDOPPLER)                                          DOPP013
      ENTRY ELECT1                                                      DOPP014
      IF (READFN.EQ.0.) GO TO 10                                        DOPP015
      READFN=0.                                                         DOPP016
      READ 1000, NOC,(HPC(I),FN2C(I),I=1,NOC)                           DOPP017
 1000 FORMAT (I4/(F8.2,E12.4))                                          DOPP018
      PRINT 1200, (HPC(I),FN2C(I),I=1,NOC)                              DOPP019
 1200 FORMAT(1H1,14X,6HHEIGHT,4X,16H      DN/DT       /(1X,F20.10,E20.10))DOPP020
      A=0.                                                              DOPP021
      IF(FN2C(1).NE.0.) A=ALOG(FN2C(2)/FN2C(1))/(HPC(2)-HPC(1))         DOPP022
      FN2C(1)=K*FN2C(1)                                                 DOPP023
      FN2C(2)=K*FN2C(2)                                                 DOPP024
      SLOPE(1)=A*FN2C(1)                                                DOPP025
      SLOPE(NOC)=0.                                                     DOPP026
      NMAX=1                                                            DOPP027
      DO 6 I=2,NOC                                                      DOPP028
      IF (FN2C(I).GT.FN2C(NMAX)) NMAX=I                                 DOPP029
      IF (I.EQ.NOC) GO TO 4                                             DOPP030
      FN2C(I+1)=K*FN2C(I+1)                                             DOPP031
      DO 3 J=1,3                                                        DOPP032
      M=I+J-2                                                           DOPP033
      MAT(J,1)=1.                                                       DOPP034
      MAT(J,2)=HPC(M)                                                   DOPP035
      MAT(J,3)=HPC(M)**2                                                DOPP036
    3 MAT(J,4)=FN2C(M)                                                  DOPP037
      CALL GAUSEL (MAT,4,3,4,NRANK)                                     DOPP038
      IF (NRANK.LT.3) GO TO 60                                          DOPP039
      SLOPE(I)=MAT(2,4)+2.*MAT(3,4)*HPC(I)                              DOPP040
    4 DO 5 J=1,2                                                        DOPP041
      M=I+J-2                                                           DOPP042
      MAT(J,1)=1.                                                       DOPP043
      MAT(J,2)=HPC(M)                                                   DOPP044
      MAT(J,3)=HPC(M)**2                                                DOPP045
      MAT(J,4)=HPC(M)**3                                                DOPP046
      MAT(J,5)=FN2C(M)                                                  DOPP047
      L=J+2                                                             DOPP048
      MAT(L,1)=0.                                                       DOPP049
      MAT(L,2)=1.                                                       DOPP050
      MAT(L,3)=2.*HPC(M)                                                DOPP051
      MAT(L,4)=3.*HPC(M)**2                                             DOPP052
    5 MAT(L,5)=SLOPE(M)                                                 DOPP053
      CALL GAUSEL (MAT,4,4,5,NRANK)                                     DOPP054
      IF (NRANK.LT.4) GO TO 60                                          DOPP055
      ALPHA(I)=MAT(1,5)                                                 DOPP056
      BETA(I)=MAT(2,5)                                                  DOPP057
      GAMMA(I)=MAT(3,5)                                                 DOPP058
    6 DELTA(I)=MAT(4,5)                                                 DOPP059
      HMAX=AMAX1(HMAX,HPC(NMAX))                                        DOPP060
      NH=2                                                              DOPP061
   10 H=R(1)-EARTHR                                                     DOPP062
      F2=F*F                                                            DOPP063
      IF (H.GE.HPC(1)) GO TO 12                                         DOPP064
   11 NH=2                                                              DOPP065
```

```
      IF(FN2C(1).EQ.0.) GO TO 50                                        DOPP066
      X=FN2C(1)*EXP(A*(H-HPC(1)))/F2                                    DOPP067
      GO TO 50                                                          DOPP068
   12 IF (H.GE.HPC(NOC)) GO TO 18                                       DOPP069
      NSTEP=1                                                           DOPP070
      IF (H.LT.HPC(NH-1)) NSTEP=-1                                      DOPP071
   15 IF (HPC(NH-1).LE.H.AND.H.LT.HPC(NH)) GO TO 16                     DOPP072
      NH=NH+NSTEP                                                       DOPP073
      GO TO 15                                                          DOPP074
   16 X=(ALPHA(NH)+H*(BETA(NH)+H*(GAMMA(NH)+H*DELTA(NH))))/F2           DOPP075
      GO TO 50                                                          DOPP076
   18 X=FN2C(NOC)/F2                                                    DOPP077
   50 CONTINUE                                                          DOPP078
      RETURN                                                            DOPP079
   60 PRINT 6000, I,HPC(I)                                              DOPP080
 6000 FORMAT(4H THE,I4,55HTH POINT IN THE   DN/DT          PROFILE HAS TDOPP081
     1HE HEIGHT,F8.2,40H KM, WHICH IS THE SAME AS ANOTHER POINT.)       DOPP082
      CALL EXIT                                                         DOPP083
         END                                                            DOPP084
                                                                        DOPP085
```

APPENDIX 5. MODELS OF THE EARTH'S MAGNETIC FIELD WITH INPUT PARAMETER FORMS

The following models of the earth's magnetic field are available. The input parameter forms, which describe the model, and the subroutine listings are given on the pages shown.

a.	Constant dip and gyrofrequency (CONSTY)	142
b.	Earth-centered dipole (DIPOLY)	143
c.	Constant dip. Gyrofrequency varies as the inverse cube of the distance from the center of the earth (CUBEY)	144
d.	Spherical harmonic expansion (HARMONY)	145

To add other models of the earth's magnetic field the user must write a subroutine that will calculate the normalized strength and direction of the earth's magnetic field (Y, Y_r, Y_θ, Y_φ) and their gradients ($\partial Y/\partial r$, $\partial Y/\partial\theta$, $\partial Y/\partial\varphi$, $\partial Y_r/\partial r$, $\partial Y_r/\partial\theta$, $\partial Y_r/\partial\varphi$, $\partial Y_\theta/\partial r$, $\partial Y_\theta/\partial\theta$, $\partial Y_\theta/\partial\varphi$, $\partial Y_\varphi/\partial r$, $\partial Y_\varphi/\partial\theta$, $\partial Y_\varphi/\partial\varphi$) as a function of position in spherical polar coordinates (r, θ, φ). ($Y = f_H/f$, where f_H is the electron gyrofrequency and f is the wave frequency.)

The restrictions on electron density models also apply to models of the earth's magnetic field. The coordinates r, θ, φ refer to the computational coordinate system, which is not necessarily the same as geographic coordinates. W24 and W25 give the geographic latitude and longitude of the north pole of the computational coordinate system.

The input to the subroutine (r, θ, ϕ) is through blank common. (See Table 3.) The output is through common block /YY/. (See Table 9.) It is useful if the name of the subroutine suggests the model to which it corresponds. It should have an entry point MAGY so that other subroutines in the program can call it. Any parameters needed by the subroutine should be input into W201 through W249 of the W array. (See Table 2.) If the subroutine needs massive amounts of data, these should be read in by the subroutine following the example of subroutine HARMONY.

INPUT PARAMETER FORM FOR SUBROUTINE CONSTY

An ionospheric model of the earth's magnetic field consisting of constant dip and gyrofrequency

Specify:

gyrofrequency, f_H = _____ MHz (W201)

dip, I = _____ degrees (W202)
radians

The magnetic meridian is defined by the geographic coordinates of the north magnetic pole:

latitude = _____ radians
degrees north (W24)

longitude = _____ radians
degrees east (W25)

```
      SUBROUTINE CONSTY                                                 CONY001
C         CONSTANT DIP AND GYROFREQUENCY                                CONY002
      COMMON /YY/ MODY,Y,PYPR,PYPTH,PYPPH,YR,PYRPR,PYRPT,PYRPP,YTH,PYTPRCONY003
     1,PYTPT,PYTPP,YPH,PYPPR,PYPPT,PYPPP                                CONY004
      COMMON /WW/ ID(10),W0,W(400)                                      CONY005
      EQUIVALENCE (F,W(6)),(FH,W(201)),(DIP,W(202))                     CONY006
      DATA (MODY=6HCONSTY)                                              CONY007
      ENTRY MAGY                                                        CONY008
      Y=FH/F                                                            CONY009
      YR=Y*SIN(DIP)                                                     CONY010
      YTH=Y*COS(DIP)                                                    CONY011
      RETURN                                                            CONY012
         END                                                            CONY013-
```

INPUT PARAMETER FORM FOR SUBROUTINE DIPOLY

An ionospheric model of the earth's magnetic field consisting of an earth centered dipole

The gyrofrequency is given by:

$$f_H = f_{H_0} \left(\frac{R_0 + h}{R_0}\right)^3 \left(1 + 3\cos^2\lambda\right)^{\frac{1}{2}}$$

The magnetic dip angle, I, is given by

$\tan I = 2 \cot \lambda$

h is the height above the ground

R_0 is the radius of the earth

λ is the geomagnetic colatitude

Specify:

the gyrofrequency at the equator on the ground, f_{H_0} = _____ MHz (W201)

the geographic coordinates of the north magnetic pole

latitude = _____ radians / degrees north (W24)

longitude = _____ radians / degrees east (W25)

```
      SUBROUTINE DIPOLY                                                 DIP0001
      COMMON /CONST/ PI,PIT2,PID2,DUM(5)                                DIP0002
      COMMON /YY/ MODY,Y,PYPR,PYPTH,PYPPH,YR,PYRPR,PYRPT,PYRPP,YTH,PYTPRDIP0003
     1,PYTPT,PYTPP,YPH,PYPPR,PYPPT,PYPPP                                DIP0004
      COMMON R(6) /WW/ ID(10),W0,W(400)                                 DIP0005
      EQUIVALENCE (EARTHR,W(2)),(F,W(6)),(FH,W(201))                    DIP0006
      DATA (MODY=6HDIPOLY)                                              DIP0007
      ENTRY MAGY                                                        DIP0008
      SINTH=SIN(R(2))                                                   DIP0009
      COSTH=SIN(PID2-R(2))                                              DIP0010
      TERM9=SQRT(1.+3.*COSTH**2)                                        DIP0011
      T1=FH*(EARTHR/R(1))**3/F                                          DIP0012
      Y=T1*TERM9                                                        DIP0013
      YR= 2.*T1*COSTH                                                   DIP0014
      YTH= T1*SINTH                                                     DIP0015
      PYRPR=-3.*YR/R(1)                                                 DIP0016
      PYRPT=-2.*YTH                                                     DIP0017
      PYTPR=-3.*YTH/R(1)                                                DIP0018
      PYTPT=.5*YR                                                       DIP0019
      PYPR=-3.*Y/R(1)                                                   DIP0020
      PYPTH=-3.*Y*SINTH*COSTH/TERM9**2                                  DIP0021
      RETURN                                                            DIP0022
         END                                                            DIP0023-
```

INPUT PARAMETER FORM FOR SUBROUTINE CUBEY

A model of the earth's magnetic field consisting of a constant dip and a gyrofrequency which varies as the inverse cube of the distance from the center of the earth

This model has the same height variation as a dipole magnetic field.

The gyrofrequency is given by:

$$f_H = f_{Ho}\left(\frac{a}{r}\right)^3$$

a is the radius of the earth.

r is the distance from the center of the earth.

Specify:

gyrofrequency at the ground, f_{Ho} = _____ MHz (W201)

dip, I = _____ radians / degrees (W202)

The magnetic meridian is defined by the geographic coordinates of the north magnetic pole:

latitude = _____ radians / degrees north / km (W24)

longitude = _____ radians / degrees east / km (W25)

```
      SUBROUTINE CUBEY                                                  CUBE001
C       CONSTANT DIP.                                                   CUBE002
C       GYROFREQ DECREASES AS CUBE OF DISTANCE FROM CENTER OF EARTH.    CUBE003
C       THIS MODEL HAS SAME HEIGHT VARIATION AS A DIPOLE FIELD.         CUBE004
      COMMON /YY/ MODY,Y,PYPR,PYPTH,PYPPH,YR,PYRPR,PYRPT,PYRPP,YTH,PYTPRCUBE005
     1,PYTPT,PYTPP,YPH,PYPPR,PYPPT,PYPPP                                CUBE006
      COMMON R /WW/ ID(10),WO,W(400)                                    CUBE007
      EQUIVALENCE (EARTHR,W(2)),(F,W(6)),(FH,W(201)),(DIP,W(202))       CUBE008
      DATA(MODY=5HCUBEY)                                                CUBE009
      ENTRY MAGY                                                        CUBE010
       Y=(EARTHR/R)**3 *FH/F                                            CUBE011
      YR= Y*SIN(DIP)                                                    CUBE012
      YTH= Y*COS(DIP)                                                   CUBE013
      PYPR=-3.*Y/R                                                      CUBE014
      PYRPR=-3.*YR/R                                                    CUBE015
      PYTPR=-3.*YTH/R                                                   CUBE016
      RETURN                                                            CUBE017
         END                                                            CUBE018-
```

INPUT PARAMETER FORM FOR SUBROUTINE HARMONY

A model of the earth's magnetic field based on a spherical harmonic expansion

The upward, southerly, and easterly components of the earth's magnetic field are given by:

$$H_r = -\sum_{n=o}^{6} (n+1)\left(\frac{a}{r}\right)^{n+2} \sum_{m=o}^{n} H_n^{\ m}(\theta)\left(g_n^m \cos m\varphi + h_n^m \sin m\varphi\right)$$

$$H_\theta = -\frac{1}{\sin\theta}\sum_{n=o}^{6}\left(\frac{a}{r}\right)^{n+2} \sum_{m=o}^{n} G_n^{\ m}(\theta)\left(g_n^m \cos m\varphi + h_n^m \sin m\varphi\right)$$

$$H_\varphi = \frac{1}{\sin\theta}\sum_{n=o}^{6}\left(\frac{a}{r}\right)^{n+2} \sum_{m=o}^{n} m\, H_n^{\ m}(\theta)\left(h_n^m \cos m\varphi - g_n^m \sin m\varphi\right)$$

where

a is the radius of the earth.

r, θ, φ are spherical (earth-centered) polar coordinates.

$$H_o^{\ o}(\theta) = 1$$

$$H_1^{\ o}(\theta) = \cos\theta$$

$$H_1^{\ 1}(\theta) = \sin\theta$$

$$H_{m+1}^{\ m}(\theta) = H_m^{\ m}(\theta)\cos\theta$$

$$H_{m+1}^{\ m+1}(\theta) = H_m^{\ m}(\theta)\sin\theta$$

$$H_{n+2}^{\ m}(\theta) = H_{n+1}^{\ m}(\theta)\cos\theta - \frac{(n+m+1)(n-m+1)}{(2n+3)\quad(2n+1)} H_n^{\ m}(\theta)$$

$$G_n^{\ m}(\theta) = -\frac{d}{d\theta} H_n^{\ m}(\theta)\sin\theta$$

$$G_m^{\ m}(\theta) = -m H_m^{\ m}(\theta) \cos\theta$$

$$G_{n+1}^{\ m}(\theta) = -(n+1) H_{n+1}^{\ m}(\theta)\cos\theta + \frac{(n+m+1)(n-m+1)}{2n+1} H_n^{\ m}(\theta)$$

The recursion formulas for calculating $H_n^{\ m}(\theta)$ and $G_n^{\ m}(\theta)$ are from Eckhouse (1964).

This subroutine uses coefficients $g_n^{\ m}$ and $h_n^{\ m}$ for Gauss normalization. Some coefficients are now being published for Schmidt normalization (e. g. Cain and Sweeney, 1970). The factors $S_{n,m}$ used for converting the "Schmidt normalized" coefficients to the "Gauss normalized" coefficients are as follows (Cain, et. al., 1968, Chapman and Bartels, 1940):

$$S_{o,o} = -1$$

$$S_{n,o} = S_{n-1,o}\left[\frac{2n-1}{n}\right]$$

$$S_{n,1} = S_{n,o}\sqrt{\frac{2n}{n+1}}$$

$$S_{n,m} = S_{n,m-1}\sqrt{\frac{(n-m+1)}{n+m}} \qquad \text{for } m>1$$

By convention, the "Gauss normalized" coefficient $g_1^{\ o}$ is positive, whereas the "Schmidt normalized" coefficient $g_1^{\ o}$ is negative. Coefficients based on more recent data on the earth's magnetic field including more satellite data are in the POGO 8/69 model.

Specify below the Gauss coefficients $g_n^{\ m}$ and $h_n^{\ m}$ in gauss.

	columns 2 → 10	columns 11 → 20	columns 21 → 30	columns 31 → 40	columns 41 → 50	columns 51 → 60	columns 61 → 70
1st card	$g_o^{\ o}$ = _____						
2nd card	$g_1^{\ o}$ = _____	g_1^1= _____					
3rd card	$g_2^{\ o}$ = _____	g_2^1= _____	g_2^2= _____				
4th card	$g_3^{\ o}$ = _____	g_3^1= _____	g_3^2= _____	g_3^3= _____			
5th card	$g_4^{\ o}$ = _____	g_4^1= _____	g_4^2= _____	g_4^3= _____	g_4^4= _____		
6th card	$g_5^{\ o}$ = _____	g_5^1= _____	g_5^2= _____	g_5^3= _____	g_5^4= _____	g_5^5= _____	
7th card	$g_6^{\ o}$ = _____	g_6^1= _____	g_6^2= _____	g_6^3= _____	g_6^4= _____	g_6^5= _____	g_6^6= _____

8th card	h_o^o = _____						
9th card	h_1^o = _____	h_1^1= _____					
10th card	h_2^o = _____	h_2^1= _____	h_2^2= _____				
11th card	h_3^o = _____	h_3^1= _____	h_3^2= _____	h_3^3= _____			
12th card	h_4^o = _____	h_4^1= _____	h_4^2= _____	h_4^3= _____	h_4^4= _____		
13th card	h_5^o = _____	h_5^1= _____	h_5^2= _____	h_5^3= _____	h_5^4= _____	h_5^5= _____	
14th card	h_6^o = _____	h_6^1= _____	h_6^2= _____	h_6^3= _____	h_6^4= _____	h_6^5= _____	h_6^6= _____

Set W200 = 1. to read in a set of coefficients.

This subroutine represents:

$H_n^m(\theta)$ by H(m+1, n+1)

$G_n^m(\theta)$ by G(m+1, n+1)

g_n^m by GG(m+1, n+1)

h_n^m by HH(m+1, n+1)

```
      SUBROUTINE HARMONY                                                HARM001
C     MODEL OF THE EARTH S MAGNETIC FIELD BASED ON A HARMONIC ANALYSIS  HARM002
      DIMENSION PHPTH(7,7),PGPTH(7,7),A1(7,7),B1(7,7)                   HARM003
      DIMENSION H(7,7),G(7,7),GG(7,7),HH(7,7),SINP(7),COSP(7)           HARM004
      COMMON /YY/ MODY,Y,PYPR,PYPTH,PYPPH,YR,PYRPR,PYRPT,PYRPP,YTH,PYTPRHARM005
     1,PYTPT,PYTPP,YPH,PYPPR,PYPPT,PYPPP                                HARM006
      COMMON R(6) /WW/ ID(10),WO,W(400)                                 HARM007
      COMMON /CONST/ PI,PIT2,PID2,DUM(5)                                HARM008
      EQUIVALENCE (THETA,R(2)),(PHI,R(3))                               HARM009
      EQUIVALENCE (EARTHR,W(2)),(F,W(6)),(READFH,W(200))                HARM010
C     RATIO OF CHARGE TO MASS FOR FLECTRON                              HARM011
      DATA(EOM=1.7589E7)                                                HARM012
      DATA (SET=0.),(H=1.,48(0.)),(G=49(0.)),(PHPTH=49(0.))             HARM013
     1 ,(PGPTH=49(0.)),(MODY=7HHARMONY)                                 HARM014
      ENTRY MAGY                                                        HARM015
      IF(SET) GO TO 2                                                   HARM016
      DO 1 M=1,7                                                        HARM017
      DO 1 N=1,7                                                        HARM018
      B1(M,N)=(N+M-1)*(N-M+1)/(2*N-1.)                                  HARM019
    1 A1(M,N)=B1(M,N)/(2*N+1)                                           HARM020
      SET=1.                                                            HARM021
```

```
    2 IF(READFH.EQ.0.) GO TO 3                                          HARM022
      READ 2000,GG,HH                                                   HARM023
 2000 FORMAT(1X,F9.4,6F10.4)                                            HARM024
      PRINT 2100,GG                                                     HARM025
 2100 FORMAT(1H1,10X,1H0,14X,1H1,14X,1H2,14X,1H3,14X,1H4,14X,1H5,14X,1H6HARM026
     1 /9X,7(1HG,14X)/10X,7(1HN,14X)//(1X,7F15.6))                      HARM027
      PRINT 2200,HH                                                     HARM028
 2200 FORMAT(//  11X,1H0,14X,1H1,14X,1H2,14X,1H3,14X,1H4,14X,1H5,14X,1H6HARM029
     1 /9X,7(1HH,14X)/10X,7(1HN,14X)//(1X,7F15.6))                      HARM030
      READFH=0.                                                         HARM031
    3 COSTHE=COS(THETA)                                                 HARM032
      SINTHE=SIN(THETA)                                                 HARM033
      AOR=EARTHR/R(1)                                                   HARM034
      PAORPR=-AOR/R(1)                                                  HARM035
      CNST2=AOR                                                         HARM036
      PCNSPR=PAORPR                                                     HARM037
      FIN1=PFIN1R=PFIN1T=PFIN1P=0.                                      HARM038
      FIN2=PFIN2R=PFIN2T=PFIN2P=0.                                      HARM039
      FIN3=PFIN3R=PFIN3T=PFIN3P=0.                                      HARM040
      DO 4 M=1,7                                                        HARM041
      SINP(M)=SIN((M-1)*PHI)                                            HARM042
    4 COSP(M)=COS((M-1)*PHI)                                            HARM043
      H(1,2)=COSTHE                                                     HARM044
      H(2,2)=SINTHE                                                     HARM045
      DO 5 M=1,5                                                        HARM046
      H(M+1,M+2)=COSTHE*H(M+1,M+1)                                      HARM047
      H(M+2,M+2)=SINTHE*H(M+1,M+1)                                      HARM048
      DO 5 N=M,5                                                        HARM049
    5 H(M,N+2)=COSTHE*H(M,N+1)-A1(M,N)*H(M,N)                           HARM050
      DO 6 M=1,6                                                        HARM051
      G(M+1,M+1)=-M*COSTHE*H(M+1,M+1)                                   HARM052
      PHPTH(M+1,M+1)=-G(M+1,M+1)/SINTHE                                 HARM053
      PGPTH(M+1,M+1)=M*SINTHE*H(M+1,M+1)-M*COSTHE*PHPTH(M+1,M+1)        HARM054
      DO 6 N=M,6                                                        HARM055
      G(M,N+1)=-N*COSTHE*H(M,N+1)+B1(M,N)*H(M,N)                        HARM056
      PHPTH(M,N+1)=-G(M,N+1)/SINTHE                                     HARM057
    6 PGPTH(M,N+1)=N*SINTHE*H(M,N+1)-N*COSTHE*PHPTH(M,N+1)+B1(M,N)*PHPTHHARM058
     1 (M,N)                                                            HARM059
      DO 8 N=1,7                                                        HARM060
      CR=PCRPTH=PCRPPH=0.                                               HARM061
      CTH=PCTHPT=PCTHPP=0.                                              HARM062
      CPH=PCPHPT=PCPHPP=0.                                              HARM063
      DO 7 M=1,N                                                        HARM064
      TEMP1=GG(M,N)*COSP(M)+HH(M,N)*SINP(M)                             HARM065
      TEMP2=(M-1)*(HH(M,N)*COSP(M)-GG(M,N)*SINP(M))                     HARM066
      CR    =CR    +H(M,N)*TEMP1                                        HARM067
      PCRPTH=PCRPTH+PHPTH(M,N)*TEMP1                                    HARM068
      PCRPPH=PCRPPH+H(M,N)*TEMP2                                        HARM069
      CTH    =CTH    +G(M,N)*TEMP1                                      HARM070
      PCTHPT=PCTHPT+PGPTH(M,N)*TEMP1                                    HARM071
      PCTHPP=PCTHPP+G(M,N)*TEMP2                                        HARM072
      CPH    =CPH    +H(M,N)*TEMP2                                      HARM073
      PCPHPT=PCPHPT+PHPTH(M,N)*TEMP2                                    HARM074
    7 PCPHPP=PCPHPP-H(M,N)*(M-1)**2*TEMP1                               HARM075
      CNST2=CNST2*AOR                                                   HARM076
      PCNSPR=CNST2*PAORPR+AOR*PCNSPR                                    HARM077
      FIN1=FIN1+N*CNST2*CR                                              HARM078
      PFIN1R=PFIN1R+N*PCNSPR*CR                                         HARM079
      PFIN1T=PFIN1T+N*CNST2*PCRPTH                                      HARM080
      PFIN1P=PFIN1P+N*CNST2*PCRPPH                                      HARM081
      FIN2=FIN2+CNST2*CTH                                               HARM082
      PFIN2R=PFIN2R+PCNSPR*CTH                                          HARM083
      PFIN2T=PFIN2T+CNST2*PCTHPT                                        HARM084
      PFIN2P=PFIN2P+CNST2*PCTHPP                                        HARM085
      FIN3=FIN3+CNST2*CPH                                               HARM086
```

```
      PFIN3R=PFIN3R+PCNSPR*CPH                                          HARM087
      PFIN3T=PFIN3T+CNST2*PCPHPT                                        HARM088
    8 PFIN3P=PFIN3P+CNST2*PCPHPP                                        HARM089
      HTHETA=-FIN2/SINTHE                                               HARM090
      HPHI=FIN3/SINTHE                                                  HARM091
C********* CONVERT FROM MAG FIELD IN GAUSS TO GYROFREQ IN MHZ           HARM092
      CONST=-EOM/PIT2*1.E-6/F                                           HARM093
      YR=-CONST*FIN1                                                    HARM094
      YTH=CONST*HTHETA                                                  HARM095
      YPH=CONST*HPHI                                                    HARM096
      Y=SQRT(YR**2+YTH**2+YPH**2)                                       HARM097
      PYRPR=-CONST*PFIN1R                                               HARM098
      PYTPR=-CONST*PFIN2R/SINTHE                                        HARM099
      PYPPR=CONST*PFIN3R/SINTHE                                         HARM100
      PYPR=(YR*PYRPR+YTH*PYTPR+YPH*PYPPR)/Y                             HARM101
      PYRPT=-CONST*PFIN1T                                               HARM102
      PYTPT=-CONST*(PFIN2T/SINTHE+HTHETA*COSTHE/SINTHE)                 HARM103
      PYPPT=CONST*(PFIN3T/SINTHE-HPHI*COSTHE/SINTHE)                    HARM104
      PYPTH=(YR*PYRPT+YTH*PYTPT+YPH*PYPPT)/Y                            HARM105
      PYRPP=-CONST*PFIN1P                                               HARM106
      PYTPP=-CONST*PFIN2P/SINTHE                                        HARM107
      PYPPP=CONST*PFIN3P/SINTHE                                         HARM108
      PYPPH=(YR*PYRPP+YTH*PYTPP+YPH*PYPPP)/Y                            HARM109
      RETURN                                                            HARM110
C   COEFFICIENTS IN GAUSSIAN UNITS FROM JONES AND MELOTTE (1953).       HARM111
C   THE FOLLOWING 14 CARDS CAN BE USED AS DATA CARDS FOR THIS SUBROUTINEHARM112
C   0.                                                                  HARM113
C.3039    .0218                                                         HARM114
C.0176   -.0509    -.0135                                               HARM115
C-.0255   .0515    -.0236    -.0074                                     HARM116
C-.0393  -.0397    -.0238     .0087    -.0018                           HARM117
C.0293   -.0329    -.0130     .0031     .0034     .0005                 HARM118
C-.0211  -.0073    -.0007     .0210     .0017    -.0004     .0006       HARM119
C 0.                                                                    HARM120
C        -.0555                                                         HARM121
C         .0260    -.0044                                               HARM122
C         .0190    -.0033    -.0001                                     HARM123
C        -.0139     .0076     .0019     .0010                           HARM124
C         .0057    -.0018     .0009     .0032    -.0004                 HARM125
C        -.0026    -.0204     .0018     .0009     .0004     .0002       HARM126
C   THE FOLLOWING SET OF GAUSS NORMALIZED COEFFICIENTS WERE CONVERTED   HARM127
C   FROM THE SCHMIDT NORMALIZED COEFFICIENTS CALCULATED BY LINEARLY     HARM128
C   EXTRAPOLATING TO EPOCH 1974 THE COEFFICIENTS PUBLISHED FOR EPOCH    HARM129
C  1960 BY CAIN AND SWEENEY (1970).  (USES EARTH RADIUS = 6371.2)       HARM130
C .000000                                                               HARM131
C+.300953 +.020298                                                      HARM132
C+.028106 -.05214   -.014435                                            HARM133
C-.0308   +.06560   -.025252  -.006952                                  HARM134
C-.041243 -.043956  -.016897  +.008021  -.002525                        HARM135
C+.014742 -.037078  -.018906  +.002819  +.003656  +.000036              HARM136
C-.006713 -.012234  -.004364  +.02137   +.001593  -.000072  +.00068     HARM137
C .000000                                                               HARM138
C .000000 -.057886                                                      HARM139
C .000000 +.035942  +.001129                                            HARM140
C .000000 +.011084  -.004421  +.001180                                  HARM141
C .000000 -.010299  +.008794  -.000086  +.002256                        HARM142
C .000000 -.003849  -.012615  +.007845  +.002207  -.000328              HARM143
C .000000 +.003157  -.012670  -.009281  +.002286  -.000135  +.000243    HARM144
         END                                                            HARM145-
```

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APPENDIX 6. COLLISION FREQUENCY MODELS WITH INPUT PARAMETER FORMS

The following collision frequency models are available. The input parameter forms, which describe the model, and the subroutine listings are given on the pages shown.

a.	Tabular profiles (TABLEZ)	152
b.	Constant collision frequency (CONSTZ)	155
c.	Exponential profile (EXPZ)	156
d.	Combination of two exponential profiles (EXPZ2)	157

To add other collision frequency models the user must write a subroutine that will calculate the normalized collision frequency (Z) and its gradient ($\partial Z/\partial r$, $\partial Z/\partial\theta$, $\partial Z/\partial\varphi$) as a function of position in spherical polar coordinates (r, θ, φ). ($Z = \nu/2\pi f$, where ν is the collision frequency between electrons and neutral air molecules and f is the wave frequency. If the Sen-Wyller formula for refractive index is used, then $Z = \nu_m/2\pi f$, where ν_m is the mean collision frequency.)

The restrictions on electron density models also apply to collision frequency models. The coordinates r, θ, φ refer to the computational coordinates system, which may not be the same as geographic coordinates. In particular, they are geomagnetic coordinates when the earth-centered dipole model of the earth's magnetic field is used.

The input to the subroutine (r, θ, ϕ) is through blank common. (See Table 3.) The output is through common block /ZZ/. (See Table 10.) It is useful if the name of the subroutine suggests the model to which it corresponds. It should have an entry point COLFRZ so that other subroutines in the program can call it. Any parameter needed by the subroutine should be input into W251 through W299 of the W array. (See Table 2.) If the model needs massive amounts of data, these should be read in by the subroutine following the example of subroutine TABLEZ.

INPUT PARAMETER FORM FOR SUBROUTINE TABLEZ

IONOSPHERIC COLLISION FREQUENCY PROFILE

The first card tells how many profile points in I4 format. The cards following the first card give the height and collision frequency of the profile points one point per card in F8.2, E12.4 format. The heights must be in increasing order. Set W250 = 1.0 to read in a new profile. After the cards are read, TABLEZ will reset W250 = 0.0. This subroutine makes an exponential extrapolation down using the bottom 2 points in the profile.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20		1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20	
HEIGHT h km	COLLISION FREQUENCY ν COLLISIONS/sec.	HEIGHT h km	COLLISION FREQUENCY ν COLLISIONS/sec.

```
      SUBROUTINE TABLEZ                                                 TABZ001
C     CALCULATES COLLISION FREQUENCY AND ITS GRADIENT FROM PROFILES     TABZ002
C     HAVING THE SAME FORM AS THOSE USED BY CROFTS RAY TRACING PROGRAM  TABZ003
C MAKES AN EXPONENTIAL EXTRAPOLATION DOWN USING THE BOTTOM TWO POINTS   TABZ004
C        NEEDS SUBROUTINE GAUSEL                                        TABZ005
      DIMENSION HPC(100),FN2C(100),ALPHA(100),BETA(100),                TABZ006
     1 GAMMA(100),DELTA(100),MAT(4,5),SLOPE(100)                        TABZ007
      COMMON /CONST/ PI,PIT2,PID2,DUM(5)                                TABZ008
      COMMON /ZZ/ MODZ,Z,PZPR,PZPTH,PZPPH                               TABZ009
      COMMON R(6) /WW/ ID(10),W0,W(400)                                 TABZ010
      EQUIVALENCE (EARTHR,W(2)),(F,W(6)),(READNU,W(250))                TABZ011
      REAL MAT                                                          TABZ012
      DATA (MODZ=6HTABLEZ)                                              TABZ013
      ENTRY COLFRZ                                                      TABZ014
      IF (READNU.EQ.0.) GO TO 10                                        TABZ015
      READNU=0.                                                         TABZ016
      READ 2, NOC,(HPC(I),FN2C(I),I=1,NOC)                              TABZ017
  2   FORMAT(I4/(F8.2,E12.4))                                           TABZ018
      PRINT 1200, (HPC(I),FN2C(I), I=1,NOC)                             TABZ019
 1200 FORMAT(1H1,14X,6HHEIGHT,4X,20HCOLLISION FREQUENCY   /             TABZ020
     1(1X,F20.10,E20.10))                                               TABZ021
      A=0.                                                              TABZ022
      IF(FN2C(1).NE.0.) A=ALOG(FN2C(2)/FN2C(1))/(HPC(2)-HPC(1))         TABZ023
      FN2C(1)=FN2C(1)/PIT2*1.E-6                                        TABZ024
      FN2C(2)=FN2C(2)/PIT2*1.E-6                                        TABZ025
      SLOPE(1)=A*FN2C(1)                                                TABZ026
      SLOPE(NOC)=0.                                                     TABZ027
      DO 5 I=2,NOC                                                      TABZ028
      IF(I.EQ.NOC) GO TO 6                                              TABZ029
      FN2C(I+1)= FN2C(I+1)/PIT2*1.E-6                                   TABZ030
      DO 3 J=1,3                                                        TABZ031
      M=I+J-2                                                           TABZ032
      MAT(J,1)=1.                                                       TABZ033
      MAT(J,2)=HPC(M)                                                   TABZ034
      MAT(J,3)=HPC(M)**2                                                TABZ035
  3   MAT(J,4)=FN2C(M)                                                  TABZ036
      CALL GAUSEL (MAT,4,3,4,NRANK)                                     TABZ037
      IF (NRANK.LT.3) GO TO 20                                          TABZ038
      SLOPE(I)=MAT(2,4)+2.*MAT(3,4)*HPC(I)                              TABZ039
  6   CONTINUE                                                          TABZ040
      DO 4 J=1,2                                                        TABZ041
      M=I+J-2                                                           TABZ042
      MAT(J,1)=1.                                                       TABZ043
      MAT(J,2)=HPC(M)                                                   TABZ044
      MAT(J,3)=HPC(M)**2                                                TABZ045
      MAT(J,4)=HPC(M)**3                                                TABZ046
      MAT(J,5)=FN2C(M)                                                  TABZ047
      L=J+2                                                             TABZ048
      MAT(L,1)=0.                                                       TABZ049
      MAT(L,2)=1.                                                       TABZ050
      MAT(L,3)=2.*HPC(M)                                                TABZ051
      MAT(L,4)=3.*HPC(M)**2                                             TABZ052
  4   MAT(L,5)=SLOPE(M)                                                 TABZ053
      CALL GAUSEL (MAT,4,4,5,NRANK)                                     TABZ054
      IF (NRANK.LT.4) GO TO 20                                          TABZ055
      ALPHA(I)=MAT(1,5)                                                 TABZ056
      BETA(I)=MAT(2,5)                                                  TABZ057
      GAMMA(I)=MAT(3,5)                                                 TABZ058
  5   DELTA(I)=MAT(4,5)                                                 TABZ059
      JUP=2                                                             TABZ060
   10 H=R(1)-EARTHR                                                     TABZ061
      IF (H.GE.HPC(1)) GO TO 12                                         TABZ062
   11 JUP=2                                                             TABZ063
      Z=FN2C(1)*EXP(A*(H-HPC(1)))/F                                     TABZ064
      PZPR=A*Z                                                          TABZ065
```

```
      RETURN                                                            TABZ066
   12 IF (H.GE.HPC(NOC)) GO TO 18                                       TABZ067
      NSTEP=1                                                           TABZ068
      IF (H.LT.HPC(JUP-1)) NSTEP=-1                                     TABZ069
   15 IF (HPC(JUP-1).GT.H.OR.H.GE.HPC(JUP)) GO TO 16                    TABZ070
      Z=(ALPHA(JUP)+H*(BETA(JUP)+H*(GAMMA(JUP)+H*DELTA(JUP))))/F        TABZ071
      PZPR=(BETA(JUP)+H*(2.*GAMMA(JUP)+H*3.*DELTA(JUP)))/F              TABZ072
      RETURN                                                            TABZ073
   16 JUP=JUP+NSTEP                                                     TABZ074
      IF (JUP.LT.2) GO TO 11                                            TABZ075
      IF (JUP.LT.NOC) GO TO 15                                          TABZ076
   18 JUP=NOC                                                           TABZ077
      Z=FN2C(NOC)/F                                                     TABZ078
      PZPR=0.                                                           TABZ079
      RETURN                                                            TABZ080
   20 PRINT 21, I,HPC(I)                                                TABZ081
   21 FORMAT(4H THE,I4,58HTH POINT IN THE COLLISION FREQUENCY PROFILE HATABZ082
     1S THE HEIGHT,F8.2,40H KM, WHICH IS THE SAME AS ANOTHER POINT.)    TABZ083
      CALL EXIT                                                         TABZ084
         END                                                            TABZ085
```

INPUT PARAMETER FORM FOR SUBROUTINE CONSTZ

An ionospheric collision frequency model consisting of a constant collision frequency

$\nu = 0$ for $h \leq h_{min}$

$\nu = \nu_0$ for $h > h_{min}$

Specify:

ν_0 = _____ collisions per second (W251)

h_{min} = _____ km (W252)

```
      SUBROUTINE CONSTZ                                                 CONZ001
C     CONSTANT COLLISION FREQUENCY                                      CONZ002
      COMMON /CONST/ PI,PIT2,PID2,DUM(5)                                CONZ003
      COMMON /ZZ/ MODZ,Z,PZPR,PZPTH,PZPPH                               CONZ004
      COMMON R(6) /WW/ ID(10),W0,W(400)                                 CONZ005
      EQUIVALENCE (EARTHR,W(2)),(F,W(6)),(NU,W(251)),(HMIN,W(252))      CONZ006
      REAL NU                                                           CONZ007
      DATA (MODZ=6HCONSTZ)                                              CONZ008
      ENTRY COLFRZ                                                      CONZ009
      H=R(1)-EARTHR                                                     CONZ010
      Z=0.                                                              CONZ011
      IF (H.GT.HMIN) Z=NU/(PIT2*F)*1.E-6                                CONZ012
      RETURN                                                            CONZ013
         END                                                            CONZ014-
```

INPUT PARAMETER FORM FOR SUBROUTINE EXPZ

An ionospheric collision frequency model consisting of an exponential profile

$$\nu = \nu_0 e^{-a(h-h_0)}$$

h is the height above the ground

Specify:

The collision frequency at the height h_0, ν_0 = _____ collisions per second (W251)

The reference height, h_0 = _____ km (W252)

The exponential decrease of ν with height, a = _____ km^{-1} (W253)

```
      SUBROUTINE EXPZ                                                   EXPZ001
C        EXPONENTIAL COLLISION FREQUENCY MODEL                          EXPZ002
      COMMON /CONST/ PI,PIT2,PID2,DUM(5)                                EXPZ003
      COMMON /ZZ/ MODZ,Z,PZPR,PZPTH,PZPPH                               EXPZ004
      COMMON R(6) /WW/ ID(10),W0,W(400)                                 EXPZ005
      REAL NU,NU0                                                       EXPZ006
      EQUIVALENCE (EARTHR,W(2)),(F,W(6)),(NU0,W(251)),(H0,W(252)),      EXPZ007
     1 (A,W(253))                                                       EXPZ008
      DATA (MODZ=6H EXPZ )                                              EXPZ009
      ENTRY COLFRZ                                                      EXPZ010
      H=R(1)-EARTHR                                                     EXPZ011
      NU=NU0/EXP (A*(H-H0))                                             EXPZ012
      Z=NU/(PIT2*F*1.E6)                                                EXPZ013
      PZPR    =-A*Z                                                     EXPZ014
      RETURN                                                            EXPZ015
         END                                                            EXPZ016-
```

INPUT PARAMETER FORM FOR SUBROUTINE EXPZ2

An ionospheric collision frequency model consisting of a combination of two exponential profiles

$$\nu = \nu_1 e^{-a_1(h-h_1)} + \nu_2 e^{-a_2(h-h_2)}$$

where h is the height above the ground.

Specify for the first exponential:

Collision frequency at height h_1, ν_1 = _____ collisions per second (W251)

Reference height, h_1 = _____ km (W252)

Exponential decrease of ν with height, a_1 = _____ km^{-1} (W253)

Specify for the second exponential:

Collision frequency at height h_2, ν_2 = _____ collisions per second (W254)

Reference height, h_2 = _____ km (W255)

Exponential decrease of ν with height, a_2 = _____ km^{-1} (W256)

```
      SUBROUTINE EXPZ2                                                  XPZ2001
C          COLLISION FREQUENCY PROFILE FROM TWO EXPONENTIALS            XPZ2002
      COMMON /CONST/ PI,PIT2,PID2,DUM(5)                                XPZ2003
      COMMON /ZZ/ MODZ,Z,PZPR,PZPTH,PZPPH                               XPZ2004
      COMMON R(6) /WW/ ID(10),W0,W(400)                                 XPZ2005
      EQUIVALENCE (EARTHR,W(2)),(F,W(6)),(NU1,W(251)),(H1,W(252)),      XPZ2006
     1 (A1,W(253)),(NU2,W(254)),(H2,W(255)),(A2,W(256))                 XPZ2007
      REAL NU1,NU2                                                      XPZ2008
      DATA (MODZ=6H EXPZ2)                                              XPZ2009
      ENTRY COLFRZ                                                      XPZ2010
      H=R(1)-EARTHR                                                     XPZ2011
      EXP1= NU1* EXP(-A1*(H-H1))                                        XPZ2012
      EXP2= NU2* EXP(-A2*(H-H2))                                        XPZ2013
      Z=(EXP1+EXP2)/(PIT2*F*1.E6)                                       XPZ2014
      PZPR=(-A1*EXP1-A2*EXP2)/(PIT2*F*1.E6)                             XPZ2015
      RETURN                                                            XPZ2016
         END                                                            XPZ2017-
```

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APPENDIX 7. CDC 250 PLOT PACKAGE

This appendix describes the plotting routines used by the Three-Dimensional Ray Tracing Program. The information was taken from "User's Guide to Cathode Ray Plotter Subroutines," ESSA Technical Memorandum ERLTM-ORSS 5, by L. David Lewis, January, 1970, and is printed with the permission of the author.

If you have access to a plotter, you may obtain plots by converting the following plotting commands to comparable commands on your system.

The CDC-250 Microfilm Recorder, under control of the NOAA Boulder CDC-3800 computer, plots data on the face of a high resolution cathode ray tube, which is photographed onto standard sized perforated, 35 mm film.

The plotting area, called a frame, is a square. Plotting positions are described in rectangular coordinates. Coordinate values are integers in the range 0 - 1023; (0, 0) is the "lower left hand corner".

Plotting specifications are transmitted to the plot routines via the following COMMON.

```
COMMON /DD/ IN, IOR, IT, IS, IC, ICC, IX, IY
```

The usage of each of the eight variables is listed below, followed by an explanation of the subroutine calls.

IN — Intensity.
IN=0 specifies normal intensity.
IN=1 specifies high intensity.

IOR — Orientation.
IOR=0 specifies upright orientation.
IOR=1 specifies rotated orientation (90° counter-clockwise).

IT — Italics (Font).
IT=0 specifies non-Italic (Roman) symbols.
IT=1 specifies Italic symbols.

IS Symbol size.
IS=0 specifies miniature size.
IS=1 specifies small size.
IS=2 specifies medium size.
IS=3 specifies large size.

IC Symbol case.
IC=0 specifies upper case.
IC=1 specifies lower case.

ICC Character code, 0-63 (R1 format).
ICC and IC together specify the symbol plotted.

IX X-coordinate, 0-1023.

IY Y-coordinate, 0-1023.

CALL DDINIT (N,ID) is required to initialize the plotting process.

CALL DDBP defines a vector origin at position IX, IY.

CALL DDVC plots a vector (straight line), with intensity IN, from the vector origin defined by the previous DDBP or DDVC call, to the vector end position at IX, IY. A single call to DDBP followed by successive calls to DDVC (with changing IX and IY) plots connected vectors.

CALL DDTAB initializes tabular plotting.

CALL DDTEXT (N,NT) plots a given array in a tabular mode, after initiating tabular plotting via DDTAB, as described above. NT is an array of length N, containing "text" for tabular plotting. Text consists of character codes, packed 8 per word (A8 Format). Text characters are plotted as tabular symbols until the command character ≠ (octal code 14, card code 4, 8, or the alphabetic shift counterpart of the = on the keypunch) occurs. The command character is not plotted. DDTEXT interprets the next character as a command; and after the command is processed, tabular plotting resumes until ≠ is again encountered.

≠ . means end of text: DDTEXT returns to the calling routine.

CALL DDFR causes a frame advance operation. Plotting on the current frame is completed, and the film advances to the next frame.

APPENDIX 8. SAMPLE CASE

A sample case is included with the description of the program for three reasons. First, it demonstrates the use of the program. Second, it illustrates the three types of output available (printout, punched cards, and ray path plots). Finally, it serves as a test case to verify that the user's copy of the program is running correctly. This last point is especially important if the user has had to make many modifications in converting the program to run on a computer other than a CDC 3800.

Although the ionospheric models in the sample case demonstrate the use of the program, they don't give realistic absorption for the radio waves. The absorption in the sample case is too low for two reasons. First, although the Chapman layer has a realistic electron density for the F region, it has much too low an electron density for the D region, where most of the absorption occurs. Second, the collision frequency profile in the sample case is designed for use with the Sen-Wyller formula for refractive index rather than the Appleton-Hartree formula used in the sample case. Multiplying the collision frequency profile in the sample case by 2.5 gives an effective collision frequency profile for use with the Appleton-Hartree formula that will give nearly the correct absorption for HF radio waves (Davies, 1965, p. 89).

Appendix 8a. Input Parameter Forms for the Sample Case

Filled-out input parameter forms are included to describe the sample case (i. e., show what ray paths are requested for which ionospheric models and what type of output is wanted). Furthermore, comparing them with Appendix 8b illustrates the relationship between the forms and the input data cards.

INPUT PARAMETER FORM FOR THREE-DIMENSIONAL RAY PATHS

Name _____ Project No. _____ Date _____

Ionospheric ID (3 characters) XO1

Title (75 characters) Test Case

Models:	Electron density	CHAPX	
	Perturbation	WAVE	
	Magnetic field	DIPOLY	
	Ordinary		(W1 = + 1.)
	Extraordinary	✓	(W1 = - 1.)
	Collision frequency	EXP=2	
Transmitter:	Height	0	km, nautical miles, feet (W3)
	Latitude	40	rad, (deg) km (W4)
	Longitude	-105	rad, (deg) km (W5)
	Frequency, initial	6	MHz (W7)
	final		(W8)
	step		(W9)
	Azimuth angle, initial	45	rad, (deg) clockwise of north (W11)
	final		(W12)
	step		(W13)
	Elevation angle, initial	0	rad, (deg) (W15)
	final	90	(W16)
	step	15	(W17)
Receiver:	Height	200	(km) nautical miles, feet (W20)
Penetrating rays:	Wanted	✓	(W21 = 0.)
	Not wanted		(W21 = 1.)
Maximum number of hops		3	(W22)
Maximum number of steps per hop		1000	(W23)
Maximum allowable error per step		10^{-4}	(W42)
Additional calculations:			= 1. to integrate = 2. to integrate and print
	Phase path	2	(W57)
	Absorption	2	(W58)
	Doppler shift		(W59)
	Path length		(W60)
	Other		

Printout: Every 5 steps of the ray trace (W71)

Punched cards (raysets): ✓ (W72 = 1.)

INPUT PARAMETER FORM FOR PLOTTING THE PROJECTION OF THE RAY PATH ON A VERTICAL PLANE

Coordinates of the left edge of the graph:

Latitude = 40. rad deg km north (W83)

Longitude = -105. rad deg km east (W84)

Coordinates of the right edge of the graph:

Latitude = 52.12 rad deg km north (W85)

Longitude = -81.8 rad deg km east (W86)

Height above the ground of the bottom of the graph = 0. km (W88)

Distance between tic marks = 100. rad deg km (W87)

(W81 = 1.)

INPUT PARAMETER FORM FOR PLOTTING THE PROJECTION OF THE RAY PATH ON THE GROUND

Coordinates of the left edge of the graph:

Latitude = 40. rad deg km north (W83)

Longitude = -105. rad deg km east (W84)

Coordinates of the right edge of the graph:

Latitude = 52.12 rad deg km north (W85)

Longitude = -81.8 rad deg km east (W86)

Factor to expand lateral deviation scale by = 200. (W82)

Distance between tic marks on range scale = 100. rad deg km (W87)

(W81 = 2.)

INPUT PARAMETER FORM FOR SUBROUTINE CHAPX

An ionospheric electron density model consisting of a Chapman layer with tilts, ripples, and gradients

$$f_N^2 = f_c^2 \exp\left(\alpha\left(1 - z - e^{-z}\right)\right)$$

$$z = \frac{h - h_{max}}{H}$$

$$f_c^2 = f_{co}^2 \left(1 + A \sin\left(2\pi\left(\theta - \frac{\pi}{2}\right)/B\right) + C\left(\theta - \frac{\pi}{2}\right)\right)$$

$$h_{max} = h_{max\,o} + E\left(\theta - \frac{\pi}{2}\right) R_o$$

f_N is the plasma frequency

h is the height above the ground

R_o is the radius of the earth in km

and θ is the colatitude in radians.

Specify:

Critical frequency at the equator, f_{co} = 6.5 MHz (W101)

Height of the maximum electron density at the equator, $h_{max\,o}$ = 300 km (W102)

Scale height, H = 62. km (W103)

α = 0.5 (W104, 0.5 for an α Chapman layer, 1.0 for a β Chapman layer)

Amplitude of periodic variation of f_c^2 with latitude, A = 0. (W105)

Period of variation of f_c^2 with latitude, B = 0. rad / deg / km (W106)

Coefficient of linear variation of f_c^2 with latitude, C = 0. rad^{-1} (W107)

Tilt of the layer, E = 0. rad / deg (W108)

INPUT PARAMETER FORM FOR SUBROUTINE WAVE

A perturbation to an ionospheric electron density model consisting of a "gravity-wave" irregularity traveling from north pole to south pole

$$N = N_0(1 + \Delta)$$

$$\Delta = \delta \exp - [(R - R_0 - z_0)/H]^2 \cdot \cos 2\pi \left[t' + (\pi/2 - \theta)\frac{R_0}{\lambda_x} + (R - R_0)/\lambda_z\right]$$

$$\frac{\partial N}{\partial t} = \frac{-2\pi}{\lambda_x} V_x N_0 \delta \exp - [(R - R_0 - z_0)/H]^2 \cdot \sin 2\pi \left[t' + (\pi/2 - \theta)\frac{R_0}{\lambda_x} + (R - R_0)/\lambda z\right]$$

R_0 is the radius of the earth.

R, θ, φ are the spherical (earth-centered) polar coordinates (Δ is independent of φ).

N_0 (R, θ, φ) is any electron density model.

Specify:

the height of maximum wave amplitude, z_0 = 250. km (W151)

wave-amplitude "scale height," H = 100. km (W152)

wave perturbation amplitude, δ = 0.1 [0. to 1.] (W153)

horizontal trace velocity, V_x = — km/sec (W154)
(needed only if Doppler shift is calculated)

horizontal wavelength, λ_x = 100. km (W155)

vertical wavelength, λ_z = 100. km (W156)

time in wave periods, t' = 0. [0. to 1.] (W157)

INPUT PARAMETER FORM FOR SUBROUTINE DIPOLY

An ionospheric model of the earth's magnetic field consisting of an earth centered dipole

The gyrofrequency is given by:

$$f_H = f_{H_o} \left(\frac{R_o + h}{R_o}\right)^3 \left(1 + 3\cos^2\lambda\right)^{\frac{1}{2}}$$

The magnetic dip angle, I, is given by

$\tan I = 2 \cot \lambda$

h is the height above the ground

R_o is the radius of the earth

λ is the geomagnetic colatitude

Specify:

the gyrofrequency at the equator on the ground, f_{H_o} = 0.8 MHz (W201)

the geographic coordinates of the north magnetic pole

latitude = 78.5 radians degrees north (W24)

longitude = 291. radians degrees east (W25)

INPUT PARAMETER FORM FOR SUBROUTINE EXPZ 2

An ionospheric collision frequency model consisting of a double exponential profile

$$\nu = \nu_1 e^{-a_1(h-h_1)} + \nu_2 e^{-a_2(h-h_2)}$$

where h is the height above the ground.

Specify for the first exponential:

Collision frequency at height h_1, ν_1 = 3.65×10^4 collisions per second (W251)

Reference height, h_1 = 100 km (W252)

Exponential decrease of ν with height, a_1 = 0.148 km^{-1} (W253)

Specify for the second exponential:

Collision frequency at height h_2, ν_2 = 30 collisions per second (W254)

Reference height, h_2 = 140 km (W255)

Exponential decrease of ν with height, a_2 = 0.0183 km^{-1} (W256)

Appendix 8b. Listing of Input Cards for the Sample Case

```
X01  TEST CASE
  1  0.                    OF DUPLICATE W CARDS, THE LAST ONE DOMINATES
  1  -1.                   EXTRAORDINARY RAY
  3  0.                    TRANSMITTER HEIGHT, KM
  4  40.          1        TRANSMITTER LATITUDE, DEG NORTH
  5  -105.        1        TRANSMITTER LONGITUDE, DEG EAST
  7  6.0                   INITIAL FREQUENCY, MC/S
  9  0.                    DONT STEP FREQUENCY
 11  45.0         1        INITIAL AZIMUTH ANGLE, DEGS CLOCKWISE FROM NORTH POLE
 13  0.                    DONT STEP AZIMUTH ANGLE
 15  0.           1        INITIAL ELEVATION ANGLE, DEG
 16  90.0         1        FINAL ELEVATION ANGLE, DEG
 17  15.0         1        STEP IN ELEVATION ANGLE, DEG
 20   200.                 RECEIVER HEIGHT ABOVE THE EARTH, KM
 22  3.                    NUMBER OF HOPS
 57  2.                    INTEGRATE AND PRINT PHASE PATH
 58  2.                    INTEGRATE AND PRINT ABSORPTION
 71  5.0                   NUMBER OF STEPS FOR EACH PRINTING
 72  1.                    PUNCH RAYSETS
 81  1.                    PLOT PROJECTION OF RAY PATH ON A VERTICAL PLANE
 83  40.0         1        LEFT LATITUDE OF PLOT, DEG
 84  -105.        1        LEFT LONGITUDE OF PLOT, DEG
 85  52.12        1        RIGHT LATITUDE OF PLOT, DEG
 86  -81.8        1        RIGHT LONGITUDE OF PLOT, DEG
 87  100.0         1       DISTANCE BETWEEN TIC MARKS, KM
101  6.5                   CRITICAL FREQUENCY, MC/S
102  300.0                 HMAX, KM
103  62.                   SCALE HEIGHT, KM
104  0.5                        ALPHA CHAPMAN LAYER
150  1.                    CALL PERTURBATION SUBROUTINE
151  250.                  Z0, KM
152 100.                   SH, SCALE HEIGHT, KM
153  0.1                   DELTA
155  100.                  LAMBDAX, HORIZONTAL WAVELENGTH, KM
156  100.                  LAMBDAZ, VERTICAL WAVELENGTH, KM
201  0.8                   GYROFREQUENCY ON THE GROUND AT THE EQUATOR, MHZ
 24  78.5         1        ACCEPTED STANDARD LAT. OF NORTH MAGNETIC POLE, DEG NORTH
 25  291.         1        ACCEPTED STANDARD LONG. OF NORTH MAGNETIC POLE, DEG EAST
251  3.65       E4         COLLISION FREQUENCY AT H1, /SEC
252  100.0                 H1, REFERENCE HEIGHT, KM
253  .148                  A1, EXPONENTIAL DECREASE OF NU WITH HEIGHT, /KM
254  30.                   COLLISION FREQUENCY AT H2, /SEC
255  140.                  H2, REFERENCE HEIGHT, KM
256  .0183                 A2, EXPONENTIAL DECREASE OF NU WITH HEIGHT, /KM
                           (A BLANK IN COL. 1-3 ENDS THE CURRENT  W ARRAY)
X01  TEST CASE
 71   0.                   NO PERIODIC PRINTOUT
 72  0.                    DO NOT PUNCH RAYSETS
 81  2.                    PLOT PROJECTION OF RAY PATH ON THE GROUND
 82  10.0                  LATERAL DEVIATION EXPANSION FACTOR
                           (A BLANK IN COL. 1-3 ENDS THE CURRENT  W ARRAY)
```

Col. 1-3 Identification number
Col. 4-17 Data in E14.6 format
Col. 18 A 1 indicates an angle in degrees
Col. 19 A 1 indicates a central earth angle in kilometers
Col. 20 A 1 indicates a distance in nautical miles
Col. 21 A 1 indicates a distance in feet
Col. 22-24 Left for other conversions
Col. 25-80 Description of the data

Appendix 8c. Sample Printout

```
X01  TEST CASE                                                                    11/05/74
 CHAPX     WAVE     DIPOLY     EXPZ2          APPLETON-HARTREE FORMULA EXTRAORDINARY WITH COLLISIONS

INITIAL VALUES FOR THE W ARRAY -- ALL ANGLES IN RADIANS, ONLY NONZERO VALUES PRINTED

  1 -1.00000000000+000
  2  6.37000000000+003
  4  6.98131700803-001
  5 -1.83259571460+000
  7  6.00000000000+000
 11  7.85398163390-001
 16  1.57079632679+000
 17  2.61799387791-001
 20  2.00000000000+002
 22  3.00000000000+000
 23  1.00000000000+003
 24  1.37008346280+000
 25  5.07890812331+000
 41  3.00000000000+000
 42  1.00000000000-004
 43  5.00000000000+001
 44  1.00000000000+000
 45  1.00000000000+002
 46  1.00000000000-008
 47  5.00000000000-001
 57  2.00000000000+000
 58  2.00000000000+000
 71  5.00000000000+000
 72  1.00000000000+000
 81  1.00000000000+000
 83  6.98131700803-001
 84 -1.83259571460+000
 85  9.09065606124-001
 86 -1.42767932613+000
 87  1.56985871273-002
101  6.50000000000+000
102  3.00000000000+002
103  6.19999999995+001
104  5.00000000000-001
150  1.00000000000+000
151  2.50000000000+002
152  1.00000000000+002
153  1.00000000000-001
155  1.00000000000+002
156  1.00000000000+002
201  8.00000000000-001
251  3.65000000002+004
252  1.00000000000+002
253  1.48000000001-001
254  3.00000000000+001
255  1.39999999999+002
256  1.83000000002-002
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Perturbation to the electron density model

Model of the earth's magnetic field

Collision frequency model

Electron density model

X01 TEST CASE 11/05/74

CHAPX WAVE DIPOLE EXPZZ APPLETON-HARTREE FORMULA EXTRAORDINARY WITH COLLISIONS

FREQUENCY = 6.000000 MHZ, AZIMUTH ANGLE OF TRANSMISSION = 45.000000 DEG

ELEVATION ANGLE OF TRANSMISSION = 0.000000 DEG

		HEIGHT KM	RANGE KM	AZIMUTH DEVIATION XMTR DEG	AZIMUTH DEVIATION LOCAL DEG	ELEVATION XMTR DEG	ELEVATION LOCAL DEG	POLARIZATION REAL	POLARIZATION IMAG	GROUP PATH KM	PHASE PATH KM	ABSORPTION DB
0+000	XMTR	0.0000	0.0000				0.000	0.000	-1.000	0.0000	0.0000	0.0000
-3-011	ENTR ION	52.9787	-19.7210	0.000	0.000	-0.000	7.364	0.597	-1.427	823.2593	823.2593	0.0000
0+000		59.4429	-66.0660	-0.000	-0.000	-0.000	7.797	0.513	-2.072	872.2593	872.2593	0.0000
-3-011		64.9916	906.0962	-0.000	-0.000	-0.000	8.150	0.293	-2.386	912.2593	912.2593	0.0000
-3-011		73.7639	945.2593	0.000	0.000	-0.000	8.502	0.141	-2.599	952.2593	952.2593	0.0000
-2-009		85.6755	1038.9476	0.000	-0.000	-0.000	9.345	0.021	-3.208	1048.2593	1048.2592	0.0004
-2-008		99.1450	1116.6720	0.000	0.000	-0.000	10.039	0.004	-4.060	1128.2593	1128.2584	0.0014
-9-008		113.5441	1194.0737	0.000	0.000	-0.002	10.683	0.001	-5.656	1208.2593	1208.2467	0.0034
-1-006		128.6479	1271.1132	0.000	-0.001	-0.014	10.999	0.000	-7.942	1288.2593	1288.1390	0.0058
-4-006		143.3758	1347.7763	0.000	0.002	-0.078	9.958	0.000	-5.844	1368.2593	1367.5112	0.0076
-7-006		153.8536	1416.4564	0.001	0.012	-0.269	6.649	0.000	-2.873	1440.2593	1437.7718	0.0089
-9-006		157.2033	1454.4663	0.001	0.051	-0.473	3.399	0.000	-2.083	1480.2593	1476.0613	0.0097
-3-011	MIN DIST	158.1469	1491.1561	0.002	-0.029	-0.753	0.000	0.000	-1.720	1518.9022	1512.7786	0.0106
-3-011	MIN DIST	158.1469	1491.1561	0.002	-0.029	-0.753	0.000	0.000	-1.720	1518.9022	1512.7786	0.0106
-6-011	WAVE REV	156.1177	1494.9559	0.003	-0.039	-0.786	-0.341	0.000	-1.695	1522.9022	1516.5809	0.0107
-2-006		151.2539	1583.3795	0.006	0.028	-1.756	-7.558	0.000	-1.385	1615.9022	1605.6686	0.0126
3-006		138.2039	1659.7028	0.007	0.024	-2.782	-10.588	0.000	-1.318	1695.9022	1684.0907	0.0144
6-006		108.1392	1813.6452	0.006	0.016	-4.796	-10.483	0.000	-1.341	1855.9022	1843.6645	0.0191
8-006		80.8887	1959.0039	0.004	0.015	-6.536	-9.113	0.008	-1.403	2015.9022	2003.6590	0.0218
8-005	EXIT ION	49.2239	2146.5251	0.002	0.013	-8.572	-7.138	0.205	-1.093	2239.9022	2227.6590	0.0228
0+000	GRND REF	0.0000	2900.0452	-0.000	0.010	-13.042	0.738	0.001	-1.000	2955.4929	2943.2497	0.0228
0+000	ENTR ION	52.3830	3540.8051	-0.003	0.008	-15.566	7.401	-0.933	1.636	3700.7907	3688.5475	0.0228
0+000		53.4995	3544.7368	-0.003	0.008	-15.577	7.436	-0.936	1.705	3704.7907	3692.5475	0.0228
-9-008		91.1164	3892.1351	-0.003	0.008	-16.214	9.661	-0.004	1.984	3957.7907	3945.5474	0.0227
-1-006	MAX LAT	107.9072	3985.1998	-0.003	0.008	-16.435	10.475	-0.000	1.635	4053.7907	4041.5426	0.0247
-1-006	WAVE REV	107.9072	3985.1998	-0.003	0.008	-16.435	10.475	-0.000	1.635	4053.7907	4041.5426	0.0247
-4-006		119.7979	4046.9652	-0.003	0.008	-16.578	10.907	-0.000	1.765	4117.7907	4105.5117	0.0266
-5-006		140.3009	4185.0294	-0.004	0.031	-16.914	9.243	-0.000	1.868	4261.7907	4248.4412	0.0303
-5-005		156.1106	4261.1682	-0.006	0.047	-17.169	4.188	-0.000	2.901	4341.7907	4325.8067	0.0320
0+000	MIN DIST	157.9016	4305.0842	-0.007	0.076	-17.366	-0.000	-0.000	9.317	4388.0431	4369.8069	0.0330

THIS RAY CALCULATION TOOK 12.362 SEC

Height of ray point above the ground.

Great circle distance along the ground between the ray point and the transmitter.

Azimuth angle of the direction of transmission in degrees clockwise from great circle between transmitter and ray point.

Azimuth angle of the wave normal in degrees clockwise from great circle between transmitter and ray point.

Elevation angle of current ray path point at the transmitter.

Angle of the wave normal with the local horizontal.

Polarization = +1 means the electric field vector is rotating counter clockwise when looking along the ray.

$V^*/$Real part (n) -1 where V is the magnitude of the wave normal vector, and n is the complex phase refractive index. This quantity would be zero if there were no errors in the numerical integration.

X01 TEST CASE 11/05/74

CHAPX WAVE DIPOLY EXPZ2 APPLETON-HARTREE FORMULA EXTRAORDINARY WITH COLLISIONS

FREQUENCY = 6.000000 MHZ, AZIMUTH ANGLE OF TRANSMISSION = 45.000000 DEG

ELEVATION ANGLE OF TRANSMISSION = 15.000000 DEG

		HEIGHT KM	RANGE KM	AZIMUTH DEVIATION XMTR DEG	AZIMUTH DEVIATION LOCAL DEG	ELEVATION XMTR DEG	ELEVATION LOCAL DEG	POLARIZATION REAL	POLARIZATION IMAG	GROUP PATH KM	PHASE PATH KM	ABSORPTION DB
0+000	XMTR	0.0000	0.0000				15.000	0.001	-1.000	0.0000	0.0000	0.0000
0+000	ENTR ION	52.9852	186.0044	-0.000	-0.000	15.000	16.673	3.119	-3.368	194.1401	194.1401	0.0000
0+000		60.7838	211.6249	-0.000	-0.000	15.000	16.903	2.323	-7.474	221.1401	221.1401	0.0000
0+000		66.6275	230.5627	-0.000	-0.000	15.000	17.074	1.297	-9.793	241.1401	241.1401	0.0000
-5-010		79.6832	272.1042	-0.000	-0.000	15.000	17.447	0.369	-18.918	285.1401	285.1401	0.0001
-7-009		91.7890	309.7220	-0.000	0.000	15.000	17.785	0.359	-110.019	325.1401	325.1400	0.0004
-2-008		104.1175	347.1979	-0.000	-0.000	15.000	18.116	-0.015	29.112	365.1401	365.1388	0.0011
1-006		126.7922	414.2887	-0.000	0.001	14.990	18.525	-0.000	10.318	437.1401	437.0846	0.0031
3-005		151.4725	488.2463	-0.000	0.012	14.841	16.907	-0.001	459.192	517.1401	516.1469	0.0048
-9-006		161.8111	524.9617	-0.001	-0.002	14.559	13.612	0.000	-4.495	557.1401	554.2934	0.0057
-1-005		169.1102	561.6061	0.006	-0.078	14.015	8.649	0.000	-2.160	597.1401	591.1301	0.0067
-7-006		172.1418	598.0253	0.015	0.155	13.155	1.378	0.000	-1.537	637.1401	626.6153	0.0079
-3-011	MIN DIST	172.1392	604.1034	0.015	0.184	12.975	0.000	0.000	-1.481	643.8528	632.4672	0.0081
-3-011	MIN DIST	172.1392	604.1034	0.015	0.184	12.975	0.000	0.000	-1.481	643.8528	632.4672	0.0081
2-010	WAVE REV	172.0659	607.7222	0.015	0.196	12.864	-0.820	0.000	-1.452	647.8528	635.9500	0.0083
-3-005		153.8276	796.9590	0.026	-0.078	8.942	-15.755	0.000	-1.173	756.8528	735.2639	0.0114
-3-005		130.0051	790.9017	0.033	-0.036	5.835	-18.205	0.000	-1.154	836.8528	813.9074	0.0133
-3-005		105.1516	855.5089	0.037	-0.032	3.093	-17.862	0.000	-1.162	916.8528	893.8080	0.0160
-3-005		81.0547	930.6930	0.039	-0.030	0.752	-17.194	0.003	-1.174	996.8528	973.8062	0.0173
-3-005	EXIT ION	48.8393	1036.6769	0.042	-0.027	-1.983	-16.239	0.076	-1.037	1106.8528	1085.8062	0.0174
0+000	GRND REF	0.0000	1212.9251	0.046	-0.023	-5.455	14.656	-0.002	1.000	1292.1934	1269.1468	0.0174
-3-011	ENTR ION	52.9855	1403.0408	0.049	-0.020	-4.165	16.366	-1.386	1.805	1490.3081	1467.2614	0.0174
-3-011		54.1138	1406.8464	0.049	-0.020	-4.142	16.400	-1.305	2.105	1494.3081	1471.2614	0.0174
-9-007		110.0072	1565.1016	0.052	-0.018	-3.213	17.984	-0.000	2.342	1683.3081	1660.2574	0.0191
-1-006		134.9372	1659.5945	0.052	-0.018	-2.890	18.138	-0.000	2.221	1763.3081	1740.8923	0.0214
-5-005		156.1139	1726.0642	0.052	0.021	-2.688	15.432	-0.000	3.024	1835.3081	1810.6175	0.0228
-1-005		165.2114	1762.7028	0.051	-0.021	-2.675	11.173	-0.000	14.789	1875.3081	1848.1145	0.0237
-7-006		170.7131	1799.2998	0.054	-0.203	-2.779	5.562	0.000	-3.471	1915.3081	1884.3821	0.0248
0+000	MIN DIST	171.9566	1828.6204	0.059	-0.065	-2.959	-0.000	0.000	-1.952	1947.4107	1912.8923	0.0258

THIS RAY CALCULATION TOOK 11.247 SEC

X01 TEST CASE 11/05/74

CHAPX WAVE DIPOLY EXPZ2 APPLETON-HARTREE FORMULA EXTRAORDINARY WITH COLLISIONS

FREQUENCY = 6.000000 MHZ, AZIMUTH ANGLE OF TRANSMISSION = 45.000000 DEG

ELEVATION ANGLE OF TRANSMISSION = 30.000000 DEG

		HEIGHT KM	RANGE KM	AZIMUTH DEVIATION XMTR DEG	AZIMUTH DEVIATION LOCAL DEG	ELEVATION XMTR DEG	ELEVATION LOCAL DEG	POLARIZATION REAL	POLARIZATION IMAG	GROUP PATH KM	PHASE PATH KM	ABSORPTION DB
3-011	XMTR	0.0000	0.0000				30.000	-0.000	1.000	0.0000	0.0000	0.0000
0+000	ENTR ION	52.9739	89.9010	-0.000	-0.000	30.000	30.809	-0.332	1.259	104.6686	104.6686	0.0000
-3-011		60.6694	102.6627	-0.000	-0.000	30.000	30.923	-0.218	1.562	119.6686	119.6686	0.0000
0+000		65.8141	111.1535	-0.000	-0.000	30.000	31.000	-0.110	1.614	129.6686	129.6686	0.0000
-2-010		78.2077	131.4761	-0.000	-0.000	30.000	31.183	-0.017	1.689	153.6686	153.6686	0.0000
-2-009		88.5857	148.3519	-0.000	-0.000	30.000	31.334	-0.004	1.592	173.6686	173.6685	0.0002
-1-007		107.3766	178.5914	-0.000	0.000	30.000	31.600	-0.000	1.562	209.6686	209.6671	0.0008
-1-006		128.3954	211.9816	-0.000	-0.001	29.993	31.773	-0.000	1.539	249.6686	249.6257	0.0028
5-006		149.2460	245.1205	-0.002	0.009	29.935	31.083	-0.000	1.574	289.6686	289.1825	0.0029
-2-005		168.5912	277.9003	-0.011	-0.068	29.657	27.599	-0.000	1.895	329.6686	326.8860	0.0038
2-005		184.0395	310.3408	-0.010	0.029	28.909	19.956	-0.000	7.331	369.6686	368.9884	0.0052
1-006		191.3569	342.2014	-0.027	0.444	27.306	5.952	0.000	-1.911	409.6686	390.0049	0.0073
0+000	MIN DIST	191.5641	354.9408	-0.023	0.219	26.398	0.000	0.000	-1.530	425.7924	400.9614	0.0082
0+000	MIN DIST	191.5641	354.9408	-0.023	0.219	26.398	0.000	0.000	-1.530	425.7924	400.9614	0.0082
6-010	WAVE REV	191.4066	358.1065	-0.020	0.137	26.154	-1.395	0.000	-1.475	429.7924	403.6919	0.0085
4-006		172.2684	426.8860	0.135	-0.735	19.783	-23.513	0.000	-1.113	514.7924	468.9299	0.0125
2-005		135.6960	493.7148	0.235	-0.521	12.986	-29.961	0.000	-1.076	594.7924	544.6800	0.0147
2-005		95.7861	561.7275	0.298	-0.455	7.874	-29.718	0.000	-1.080	674.7924	624.5459	0.0172
2-005		56.5019	630.5940	0.348	-0.405	2.257	-29.100	0.039	-1.062	754.7924	704.5457	0.0177
2-005	EXIT ION	46.7358	644.4681	0.357	-0.397	1.406	-28.975	0.035	-1.018	770.7924	720.5457	0.0177
-3-011	GRND REF	0.0000	733.6060	0.405	-0.349	-3.299	28.173	-0.000	1.000	872.6850	822.4383	0.0177
0+000	ENTR ION	52.9781	830.3382	0.446	-0.308	-0.104	29.043	-0.277	1.223	983.3256	933.0789	0.0177
-4-007		92.6890	900.1376	0.470	-0.284	1.779	29.671	-0.002	1.497	1064.3256	1014.0788	0.0180
-2-005		132.5904	968.2234	0.490	-0.266	3.349	30.046	-0.000	1.461	1144.3256	1093.9984	0.0201
-1-005		163.3870	1021.9170	0.501	-0.223	4.356	26.978	-0.000	1.628	1208.3256	1156.1125	0.0215
-2-005		179.0552	1055.0139	0.507	-0.452	4.735	20.718	-0.000	2.556	1248.3256	1191.4144	0.0228
-9-006		188.6742	1087.7704	0.514	0.102	4.784	9.433	0.000	-4.108	1288.3256	1222.7505	0.0245
0+000	MIN DIST	189.8217	1107.5272	0.514	0.454	4.581	0.000	0.000	-1.771	1312.8736	1240.1870	0.0259

THIS RAY CALCULATION TOOK 8.642 SEC

X01 TEST CASE
CHAPX WAVE DIPOLY EXPZ2 11/05/74

APPLETON-HARTREE FORMULA EXTRAORDINARY WITH COLLISIONS

FREQUENCY = 6.000000 MHZ, AZIMUTH ANGLE OF TRANSMISSION = 45.000000 DEG

ELEVATION ANGLE OF TRANSMISSION = 45.000000 DEG

		HEIGHT KM	RANGE KM	AZIMUTH DEVIATION XMTR DEG	AZIMUTH DEVIATION LOCAL DEG	ELEVATION XMTR DEG	ELEVATION LOCAL DEG	POLARIZATION REAL	POLARIZATION IMAG	GROUP PATH KM	PHASE PATH KM	ABSORPTION DB
0+000	XMTR	0.0000	0.0000				45.000	-0.000	1.000	0.0000	0.0000	0.0000
0+000	ENTR ION	52.9759	52.3246	-0.000	-0.000	45.000	45.471	-0.113	1.100	74.6128	74.6128	0.0000
3-011		62.2439	61.3538	-0.000	-0.000	45.000	45.552	-0.059	1.207	87.6128	87.6128	0.0000
0+000		69.3926	68.2802	0.000	0.000	45.000	45.614	-0.021	1.219	97.6128	97.6128	0.0000
-3-009		85.1331	83.4659	-0.000	-0.000	45.000	45.751	-0.002	1.217	119.6128	119.6128	0.0001
-4-008		99.4743	97.2070	-0.000	-0.000	45.000	45.873	-0.000	1.213	139.6128	139.6125	0.0004
-1-007		113.8433	110.8868	-0.000	-0.000	45.000	45.986	-0.000	1.209	159.6128	159.6091	0.0009
5-005		142.5566	138.0458	-0.002	0.003	44.980	45.857	-0.000	1.209	199.6128	199.4233	0.0021
-2-005		170.1037	164.7826	-0.014	-0.106	44.789	43.268	-0.000	1.260	239.6128	237.3102	0.0030
6-005		193.2703	190.6257	-0.071	0.369	44.104	34.673	-0.000	1.601	279.6128	268.9418	0.0048
-3-011	RCVR	200.0000	200.2014	-0.132	0.607	43.622	28.480	-0.000	2.293	295.1649	278.4398	0.0058
2-006		209.6843	233.2599	-0.069	-1.590	40.436	4.780	0.000	-1.976	348.1649	304.3962	0.0095
2-006	APOGEE	209.6834	235.8629	-0.040	-1.746	40.109	3.110	0.000	-1.831	352.1649	306.3131	0.0098
3-006	WAVE REV	209.4360	241.0969	0.021	-1.982	39.431	-0.266	0.000	-1.623	360.1649	310.1892	0.0103
3-007		205.9618	259.5199	0.244	-2.001	36.819	-12.892	0.000	-1.268	388.1649	324.3166	0.0122
3-011	RCVR	200.0000	274.3768	0.389	-1.415	34.427	-23.456	0.000	-1.142	410.8543	336.8372	0.0137
1-005		172.6329	311.8736	0.547	-0.183	27.233	-41.318	0.000	-1.041	467.8543	377.6491	0.0169
4-006		117.8520	366.4800	0.568	-0.033	16.021	-45.154	0.000	-1.031	547.8543	454.5212	0.0193
6-006		89.5472	394.2849	0.570	-0.030	10.932	-44.927	0.000	-1.032	587.8543	494.5131	0.0204
6-006		38.9026	444.9524	0.574	-0.027	2.978	-44.472	0.000	-1.000	659.8543	566.5130	0.0206
6-006	EXIT ION	38.9028	444.9524	0.574	-0.027	2.978	-44.472	0.000	-1.000	659.8543	566.5130	0.0206
0+000	GRND REF	0.0000	484.7060	0.576	-0.025	-2.180	44.114	-0.000	1.000	715.5626	622.2214	0.0206
0+000	ENTR ION	52.9777	538.6621	0.578	-0.022	3.168	44.599	-0.105	1.094	791.3392	697.9980	0.0206
-6-007		110.1068	595.3574	0.581	-0.020	7.705	45.103	-0.000	1.196	872.3392	778.9960	0.0214
-6-007		138.4297	622.9739	0.581	-0.016	9.586	45.095	-0.000	1.194	912.3392	818.8774	0.0226
-4-006		165.8750	650.1889	0.579	-0.068	11.199	42.989	-0.000	1.230	952.3392	857.2445	0.0236
7-005		189.5935	676.6832	0.572	0.057	12.376	35.811	-0.000	1.434	992.3392	890.3367	0.0252
-3-011	RCVR	200.0000	691.5163	0.548	0.796	12.769	26.953	-0.000	2.159	1015.8617	905.3752	0.0266

THIS RAY CALCULATION TOOK 10.310 SEC

X01 TEST CASE 11/05/74

CHAPX WAVE DIPOLY EXPZ2 APPLETON-HARTREE FORMULA EXTRAORDINARY WITH COLLISIONS

FREQUENCY = 6.000000 MHZ, AZIMUTH ANGLE OF TRANSMISSION = 45.000000 DEG

ELEVATION ANGLE OF TRANSMISSION = 60.000000 DEG

		HEIGHT KM	RANGE KM	AZIMUTH DEVIATION XMTR DEG	AZIMUTH DEVIATION LOCAL DEG	ELEVATION XMTR DEG	ELEVATION LOCAL DEG	POLARIZATION REAL	POLARIZATION IMAG	GROUP PATH KM	PHASE PATH KM	ABSORPTION DB
0+000	XMTR	0.0000	0.0000				60.000	-0.000	1.000	0.0000	0.0000	0.0000
0+000	ENTR ION	52.9788	30.2936	-0.000	-0.000	60.000	60.272	-0.053	1.049	61.0908	61.0908	0.0000
0+000		59.9272	34.2236	-0.000	-0.000	60.000	60.308	-0.035	1.092	69.0908	69.0908	0.0000
0+000		67.7470	38.6347	-0.000	-0.000	60.000	60.348	-0.012	1.103	78.0908	78.0908	0.0000
-7-009		85.1354	48.3990	-0.000	-0.000	60.000	60.435	-0.001	1.103	98.0908	98.0908	0.0001
-1-007		102.5387	58.1107	-0.000	-0.000	60.000	60.522	-0.003	1.101	118.0908	118.0904	0.0004
-4-007		119.9525	67.7696	-0.000	-0.000	59.999	60.592	-0.000	1.100	138.0908	138.0820	0.0011
-4-006		154.5342	86.8836	-0.007	-0.005	59.967	60.149	-0.000	1.103	178.0908	177.5563	0.0022
4-005		186.1658	105.2175	-0.094	0.088	59.697	55.830	-0.000	1.158	218.0908	211.9329	0.0037
0+000	RCVR	200.0000	114.4182	-0.157	-0.851	59.325	50.578	-0.003	1.250	239.2552	225.6343	0.0050
9-005		224.6062	137.5936	-0.234	0.797	57.441	24.698	-0.000	5.656	296.2552	248.7596	0.0090
-1-004	APOGEE	225.8382	146.6930	-0.354	5.402	55.872	-5.298	0.000	-2.082	324.2552	253.2681	0.0114
-1-004	WAVE REV	225.8382	146.6930	-0.354	5.402	55.872	-5.298	0.000	-2.082	324.2552	253.2681	0.0114
-1-004		221.4155	153.9714	-0.522	9.624	54.028	-33.727	0.000	-1.188	348.2552	257.0925	0.0135
-3-005		204.2335	166.0800	-1.210	16.099	49.687	-60.227	0.000	-1.030	388.2552	269.8039	0.0168
0+000	RCVR	200.0000	168.1639	-1.383	16.876	48.744	-62.499	0.000	-1.025	395.1225	273.3158	0.0173
-1-005		153.9590	185.8715	-3.075	16.994	38.460	-69.419	0.000	-1.013	452.1225	318.1391	0.0201
-4-005		116.7265	198.8446	-4.206	15.804	29.290	-69.701	0.000	-1.013	492.1225	357.6180	0.0213
-2-005		79.2276	212.0602	-5.215	14.796	19.415	-69.586	0.000	-1.013	532.1225	397.6124	0.0223
-2-005	EXIT ION	26.7751	230.9196	-6.447	13.564	5.561	-69.411	0.000	-1.000	588.1225	453.6124	0.0223
0+000	GRND REF	0.0000	240.7118	-7.009	13.003	-1.083	69.328	-0.000	1.000	616.7330	482.2229	0.0223
-3-011	ENTR ION	52.9886	260.0662	-7.992	12.021	10.239	69.499	-0.026	1.025	673.3376	538.8275	0.0223
-3-011		56.7354	261.4245	-8.056	11.957	11.015	69.511	-0.023	1.038	677.3376	542.8275	0.0223
-3-005		113.9101	281.9789	-8.339	11.075	20.549	69.695	-0.000	1.050	738.3376	603.8243	0.0232
6-005		151.2131	295.2591	-9.446	10.601	25.514	69.477	-0.000	1.051	778.3376	643.4278	0.0244
-4-005		185.7563	308.0424	-9.910	9.231	29.335	66.600	-0.000	1.071	818.3376	678.2033	0.0259
0+000	RCVR	200.0000	313.9111	-10.115	8.555	30.682	63.564	-0.000	1.099	837.7013	691.0808	0.0271

THIS RAY CALCULATION TOOK 8.004 SEC

X01 TEST CASE 11/05/74

CHAPX WAVE DIPOLY EXPZ2 APPLETON-HARTREE FORMULA EXTRAORDINARY WITH COLLISIONS

FREQUENCY = 6.000000 MHZ, AZIMUTH ANGLE OF TRANSMISSION = 45.000000 DEG

ELEVATION ANGLE OF TRANSMISSION = 75.000000 DEG

		HEIGHT KM	RANGE KM	AZIMUTH DEVIATION XMTR DEG	AZIMUTH DEVIATION LOCAL DEG	ELEVATION XMTR DEG	ELEVATION LOCAL DEG	POLARIZATION REAL	POLARIZATION IMAG	GROUP PATH KM	PHASE PATH KM	ABSORPTION DB
0+000	XMTR	0.0000	0.0000				75.000	-0.000	1.000	0.0000	0.0000	0.0000
0+000	ENTR ION	52.9646	14.0759	-0.000	-0.000	75.000	75.127	-0.024	1.023	54.8374	54.8374	0.0000
-3-011		60.7166	16.1100	-0.000	-0.000	75.000	75.145	-0.014	1.044	62.8374	62.8374	0.0000
-3-011		65.5439	17.3768	-0.000	-0.000	75.000	75.156	-0.008	1.047	67.8374	67.8374	0.0000
-2-006		87.7850	23.1909	-0.000	-0.000	75.000	75.209	-0.000	1.048	90.8374	90.8374	0.0001
-1-007		107.1237	28.2124	-0.000	-0.000	75.000	75.253	-0.000	1.047	110.8374	110.8365	0.0005
-9-007		126.4558	33.2025	-0.001	0.000	74.999	75.279	-0.000	1.046	130.8374	130.8161	0.0013
4-007		145.6921	38.1498	-0.007	-0.012	74.994	75.188	-0.000	1.047	150.8374	150.6143	0.0019
3-005		178.7737	46.8081	-0.062	-0.321	74.920	73.849	-0.000	1.057	186.8374	183.5011	0.0030
0+000	RCVR	200.0000	52.6875	-0.362	1.334	74.783	71.357	-0.000	1.087	213.9181	202.1284	0.0046
6-005		224.9440	60.6695	-1.077	1.370	74.379	52.074	-0.000	1.568	266.9181	217.2922	0.0091
1-004		230.7996	66.1625	-1.711	-7.396	73.432	15.569	0.000	-9.825	306.9181	220.0213	0.0127
1-004	APOGEE	230.9183	67.5504	-1.511	-8.853	73.111	8.237	0.000	-3.876	314.9181	220.4673	0.0134
9-005	WAVE REV	230.2593	70.7691	-0.922	-11.181	72.307	-4.474	0.000	-1.950	330.9181	221.5782	0.0147
5-005		222.0226	84.2620	1.343	-14.206	68.508	-29.337	0.000	-1.143	378.9181	228.5798	0.0187
0+000	RCVR	200.0000	106.6297	5.511	-13.543	61.084	-46.475	0.000	-1.024	434.9673	249.2747	0.0232
2-005		161.6693	134.6131	8.542	-11.278	49.256	-55.293	0.000	-1.007	491.9673	291.7804	0.0263
-2-005		129.0155	155.6964	10.092	-9.754	38.661	-56.407	0.000	-1.006	531.9673	330.5977	0.0275
-3-006		95.7312	177.1805	11.285	-8.562	27.405	-56.282	0.000	-1.006	571.9673	370.5601	0.0289
-1-006		62.4974	196.9508	12.228	-7.620	16.463	-56.084	0.001	-1.006	611.9673	410.5608	0.0292
-1-006	EXIT ION	29.3420	220.9908	12.992	-6.856	6.551	-55.885	0.000	-1.000	651.9673	450.5608	0.0292
0+000	GRND REF	0.0000	240.7589	13.557	-6.291	-1.083	55.706	-0.000	1.000	687.4459	486.0386	0.0292
0+000	ENTR ION	52.9836	276.3403	14.370	-5.479	9.565	56.027	-0.069	1.064	751.4563	550.0489	0.0292
-8-008		87.0257	298.8468	14.784	-5.065	14.785	56.231	-0.001	1.134	792.4563	591.0489	0.0293
-6-006		120.3097	320.5884	15.128	-4.721	18.948	56.406	-0.000	1.130	832.4563	631.0396	0.0303
2-005		153.3778	342.0624	15.423	-4.425	22.353	55.904	-0.000	1.135	872.4563	670.5268	0.0314
4-005		183.3995	362.9372	15.675	-4.214	24.915	51.894	-0.000	1.195	912.4563	705.8795	0.0327
-3-011	RCVR	200.0006	375.3507	15.794	-3.644	25.987	45.376	-0.000	1.374	937.9577	722.8911	0.0343

THIS RAY CALCULATION TOOK 9.624 SEC

X01 TEST CASE 11/05/74

CHAPX WAVE DIPOLY EXPZ2 APPLETON-HARTREE FORMULA EXTRAORDINARY WITH COLLISIONS

FREQUENCY = 6.000000 MHZ, AZIMUTH ANGLE OF TRANSMISSION = 45.000000 DEG

ELEVATION ANGLE OF TRANSMISSION = 90.000000 DEG

		HEIGHT KM	RANGE KM	AZIMUTH DEVIATION XMTR DEG	AZIMUTH DEVIATION LOCAL DEG	ELEVATION XMTR DEG	ELEVATION LOCAL DEG	POLARIZATION REAL	POLARIZATION IMAG	GROUP PATH KM	PHASE PATH KM	ABSORPTION DB
0+000	XMTR	0.0000	0.0000				90.000	-0.000	1.000	0.0000	0.0000	0.0000
-3-011	ENTR ION	52.9906	0.0000			90.000	90.000	-0.000	1.009	52.9908	52.9908	0.0000
-3-011	MAX LAT	54.9906	0.0000			90.000	90.000	-0.000	1.012	54.9908	54.9908	0.0000
-3-011		56.9906	0.0000			90.000	90.000	-0.000	1.014	56.9908	56.9908	0.0000
-6-011		62.4908	0.0000			90.000	90.000	-0.000	1.018	62.4908	62.4908	0.0000
-9-011		64.9908	0.0000			90.000	90.000	-0.000	1.018	64.9908	64.9908	0.0000
-6-011		67.4908	0.0000			90.000	90.000	-0.002	1.018	67.4908	67.4908	0.0000
-6-011	WAVE REV	69.2408	0.0000			90.000	90.000	-0.002	1.019	69.2408	69.2408	0.0000
-6-011		69.2408	0.0000			90.000	90.000	-0.002	1.019	69.2408	69.2408	0.0000
-6-011		71.7408	0.0000			90.000	90.000	-0.001	1.019	71.7408	71.7408	0.0000
-3-011		74.2408	0.0000			90.000	90.000	-0.001	1.019	74.2408	74.2408	0.0000
-3-011		76.7408	0.0000			90.000	90.000	-0.001	1.019	76.7408	76.7408	0.0000
-6-011		79.2408	0.0000			90.000	90.000	-0.000	1.019	79.2408	79.2408	0.0000
-9-011		84.2408	0.0000			90.000	90.000	-0.000	1.019	84.2408	84.2408	0.0001
-2-010		89.2408	0.0000			90.000	90.000	-0.000	1.019	89.2408	89.2408	0.0001
-3-010		94.2408	0.0000	-143.742	-178.476	90.000	90.000	-0.000	1.018	94.2408	94.2407	0.0002
-1-009		105.2408	0.0000	-144.362	-179.096	90.000	90.000	-0.000	1.018	105.2408	105.2402	0.0005
-2-009		110.2339	0.0001	-145.186	-179.920	90.000	90.000	-0.000	1.018	110.2408	110.2392	0.0007
-3-009		115.2387	0.0001	-145.276	179.991	90.000	90.000	-0.000	1.018	115.2408	115.2370	0.0009
-3-009	WAVE REV	119.7365	0.0003	-145.262	0.004	90.000	90.000	-0.000	1.018	119.7408	119.7331	0.0010
-3-009		120.7358	0.0003	-145.260	0.006	90.000	90.000	-0.000	1.018	120.7408	120.7318	0.0011
-4-009		125.7304	0.0007	-145.269	-0.003	90.000	90.000	-0.000	1.018	125.7408	125.7220	0.0013
-4-009		138.6856	0.0039	-145.266	0.001	89.998	89.997	-0.000	1.018	138.7408	138.6413	0.0017
9-008		148.5779	0.0121	-145.266	0.000	89.995	89.987	-0.000	1.018	148.7408	148.4476	0.0019
3-007		158.3272	0.0318	-145.266	-0.000	89.988	89.970	-0.000	1.019	158.7408	157.9965	0.0022
3-007		167.8234	0.0700	-145.266	0.000	89.975	89.964	-0.000	1.020	168.7408	167.1041	0.0025
2-007	WAVE REV	175.1532	0.1157	-145.266	-180.000	89.961	89.993	-0.000	1.021	176.7408	173.9276	0.0029
2-007		176.9406	0.1289	-145.266	-180.000	89.957	89.970	-0.000	1.021	178.7408	175.5573	0.0029
-1-006		192.8775	0.2533	-145.266	180.000	89.922	89.409	-0.000	1.025	196.7408	188.6768	0.0048
0+000	RCVR	200.0000	0.3028	-145.266	-180.000	89.911	88.779	-0.000	1.030	207.1658	194.9665	0.0047
4-005		218.5597	0.3932	-145.266	-180.000	89.893	87.446	-0.000	1.047	236.1658	207.6856	0.0069
3-006		227.9866	0.7709	-145.266	-180.000	89.799	89.319	-0.000	1.054	256.1658	212.5942	0.0084
4-006	WAVE REV	230.2098	0.9818	-145.266	0.000	89.747	89.207	-0.000	1.052	262.1658	213.5202	0.0089
-4-004		237.3654	2.6623	-145.266	0.000	89.333	55.368	-0.000	1.017	294.1658	215.5935	0.0115
-3-003	WAVE REV	238.2305	3.6109	-145.266	0.000	89.099	-11.160	-0.000	4.505	310.1658	215.7522	0.0127
-2-003	APOGEE	238.0027	4.0933	-145.266	-0.000	88.978	-33.291	0.000	-5.606	318.1658	215.8346	0.0134
-4-004		236.0571	5.1869	-145.266	-0.000	88.696	-51.733	0.000	-1.866	334.1658	216.3578	0.0146
2-005		221.9808	9.8361	-145.266	0.000	87.375	-66.363	0.000	-1.198	374.1658	223.1020	0.0177
-6-011	RCVR	200.0000	15.3973	-145.266	0.000	85.460	-75.213	0.000	-1.080	411.0578	238.1693	0.0205
2-005		151.1549	24.7823	-145.266	0.000	80.469	-79.011	0.000	-1.042	468.0578	282.8996	0.0232
-1-005		112.0674	32.1365	-145.266	0.000	73.720	-79.110	0.000	-1.042	508.0578	322.5389	0.0245
2-006		72.7936	39.6085	-145.266	0.000	61.133	-79.045	0.002	-1.043	548.0578	362.5365	0.0252
2-006	EXIT ION	25.6748	48.6937	-145.266	0.000	27.535	-78.963	0.001	-1.000	596.0578	410.5365	0.0252
-3-011	GRND REF	0.0000	53.7019	-145.266	0.000	-0.242	78.918	-0.000	1.000	622.2176	436.6971	0.0252
-3-011	ENTR ION	52.9956	63.9944	-145.266	0.000	39.224	79.011	-0.000	1.003	676.2116	490.6911	0.0252
-1-007		85.3935	70.2016	-145.266	0.000	50.073	79.067	-0.000	1.006	709.2116	523.6910	0.0253
-2-005		124.0623	77.6423	-145.266	0.000	57.484	79.121	-0.000	1.006	749.2116	563.6739	0.0265
3-005		163.2840	84.9939	-145.266	0.000	61.819	78.758	-0.000	1.006	789.2116	602.5108	0.0277
-3-006		197.0502	92.0417	-145.266	0.000	64.210	77.455	-0.000	1.006	829.2116	633.2002	0.0297
0+000	RCVR	200.0000	92.7160	-145.266	0.000	64.368	77.233	-0.000	1.006	833.2790	635.5670	0.0300

THIS RAY CALCULATION TOOK 13.937 SEC

X01 TEST CASE 11/05/74

CHAPX WAVE DIPOLY EXPZ2 APPLETON-HARTREE FORMULA EXTRAORDINARY WITH COLLISIONS

INITIAL VALUES FOR THE W ARRAY -- ALL ANGLES IN RADIANS, ONLY NONZERO VALUES PRINTED

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  1 -1.00000000000+000
  2  5.37000000000+003
  4  6.98131700603-001
  5 -1.83259571460+000
  7  6.00000000000+000
 11  7.85398163390-001
 16  1.57079632679+000
 17  2.61799387791-001
 20  2.00000000000+002
 22  3.00000000000+000
 23  1.00000000000+003
 24  1.37008346280+000
 25  5.07890812331+000
 41  3.00000000000+000
 42  1.00000000000-004
 43  5.00000000000+001
 44  1.00000000000+000
 45  1.00000000000+002
 46  1.00000000000-008
 47  5.00000000000-001
 57  2.00000000000+000
 58  2.00000000000+000
 71  1.00000000000+003
 81  2.00000000000+000
 82  1.00000000000+001
 83  6.98131700603-001
 84 -1.83259571460+000
 85  9.09665606124-001
 86 -1.42767932813+000
 87  1.56985671273-002
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103  6.19999999995+001
104  5.00000000000-001
150  1.00000000000+000
151  2.50000000000+002
152  1.00000000000+002
153  1.00000000000-001
155  1.00000000000+002
156  1.00000000000+002
201  8.00000000000-001
251  3.65000000002+004
252  1.00000000000+002
253  1.40000000001-001
254  3.00000000000+001
255  1.33999999999+002
256  1.83000000002-002
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X01 TEST CASE 11/05/74

CHAPX WAVE DIPOLY EXPZ2 APPLETON-HARTREE FORMULA EXTRAORDINARY WITH COLLISIONS

FREQUENCY = 6.000000 MHZ, AZIMUTH ANGLE OF TRANSMISSION = 45.000000 DEG

ELEVATION ANGLE OF TRANSMISSION = 0.000000 DEG

		HEIGHT KM	RANGE KM	AZIMUTH DEVIATION XMTR DEG	AZIMUTH DEVIATION LOCAL DEG	ELEVATION XMTR DEG	ELEVATION LOCAL DEG	POLARIZATION REAL	POLARIZATION IMAG	GROUP PATH KM	PHASE PATH KM	ABSORPTION DB
0+000	XMTR	0.0000	0.0000				0.000	0.000	-1.000	0.0000	0.0000	0.0000
-3-011	ENTR ION	52.9787	818.7210	0.000	0.000	-0.000	7.364	0.597	-1.427	823.2593	823.2593	0.0000
-3-011	MIN DIST	158.1469	1491.1561	0.002	-0.029	-0.753	0.000	0.000	-1.720	1518.9022	1512.7786	0.0106
-3-011	MIN DIST	158.1469	1491.1561	0.002	-0.029	-0.753	0.000	0.000	-1.720	1518.9022	1512.7786	0.0106
-6-011	WAVE REV	158.1177	1494.9559	0.003	-0.039	-0.736	-0.341	0.000	-1.695	1522.9022	1516.5809	0.0107
8-006	EXIT ION	49.2239	2188.5251	0.003	0.013	-8.572	-7.138	0.205	-1.093	2239.9022	2227.6590	0.0220
0+000	GRND REF	0.0000	2906.0482	-0.006	0.010	-13.042	0.738	0.001	-1.000	2955.4929	2943.2497	0.0220
0+000	ENTR ION	52.9830	3640.8051	-0.003	0.008	-15.566	7.401	-0.933	1.636	3700.7907	3688.5475	0.0220
-1-006	MAX LAT	107.9072	3985.1998	-0.003	0.008	-16.435	10.475	-0.000	1.835	4053.7907	4041.5426	0.0247
-1-006	WAVE REV	107.9072	3985.1998	-0.003	0.008	-16.435	10.475	-0.000	1.835	4053.7907	4041.5426	0.0247
0+000	MIN DIST	157.9016	4305.0842	-0.007	0.076	-17.366	-0.000	-0.000	9.317	4388.0431	4369.8069	0.0330

THIS RAY CALCULATION TOOK 10.723 SEC

ELEVATION ANGLE OF TRANSMISSION = 15.000000 DEG

		HEIGHT KM	RANGE KM	AZIMUTH DEVIATION XMTR DEG	AZIMUTH DEVIATION LOCAL DEG	ELEVATION XMTR DEG	ELEVATION LOCAL DEG	POLARIZATION REAL	POLARIZATION IMAG	GROUP PATH KM	PHASE PATH KM	ABSORPTION DB
0+000	XMTR	0.0000	0.0000				15.000	0.001	-1.000	0.0000	0.0000	0.0000
0+000	ENTR ION	52.9852	186.0044	-0.000	-0.000	15.000	16.673	3.119	-3.368	194.1401	194.1401	0.0000
-3-011	MIN DIST	172.1332	604.1034	0.015	0.184	12.975	0.000	0.000	-1.481	643.8528	632.4672	0.0081
-3-011	MIN DIST	172.1332	604.1034	0.015	0.184	12.975	0.000	0.000	-1.481	643.8528	632.4672	0.0081
2-010	WAVE REV	172.0659	607.7222	0.015	0.196	12.864	-0.820	0.000	-1.452	647.8528	635.9508	0.0083
-3-005	EXIT ION	48.8383	1036.8769	0.042	-0.027	-1.983	-16.239	0.076	-1.037	1108.8528	1085.8062	0.0174
0+000	GRND REF	0.0000	1212.9251	0.046	-0.023	-5.455	14.656	-0.002	1.000	1292.1934	1269.1468	0.0174
-3-011	ENTR ION	52.9855	1403.0408	0.049	-0.020	-4.165	16.366	-1.306	1.885	1490.3081	1467.2614	0.0174
0+000	MIN DIST	171.9566	1828.6204	0.059	-0.065	-2.959	-0.000	0.000	-1.952	1947.4107	1912.8923	0.0258

THIS RAY CALCULATION TOOK 10.736 SEC

X01 TEST CASE 11/05/74

CHAPX WAVE DIPOLY EXPZ2 APPLETON-HARTREE FORMULA EXTRAORDINARY WITH COLLISIONS

FREQUENCY = 6.000000 MHZ, AZIMUTH ANGLE OF TRANSMISSION = 45.000000 DEG

ELEVATION ANGLE OF TRANSMISSION = 30.000000 DEG

	HEIGHT KM	RANGE KM	AZIMUTH DEVIATION XMTR DEG	AZIMUTH DEVIATION LOCAL DEG	ELEVATION XMTR DEG	ELEVATION LOCAL DEG	POLARIZATION REAL	POLARIZATION IMAG	GROUP PATH KM	PHASE PATH KM	ABSORPTION DB
3-011 XMTR	0.0000	0.0000				30.000	-0.000	1.000	0.0000	0.0000	0.0000
0+000 ENTR ION	52.9739	89.9010	-0.000	-0.000	30.000	30.089	-0.332	1.259	104.6686	104.6686	0.0000
0+000 MIN DIST	191.5641	354.9408	-0.023	0.219	26.398	0.000	0.000	-1.530	425.7924	400.9614	0.0082
0+000 MIN DIST	191.5641	354.9408	-0.023	0.219	26.398	0.000	0.000	-1.530	425.7924	400.9614	0.0082
6-010 WAVE REV	191.4066	358.1065	-0.020	0.137	26.154	-1.395	0.000	-1.475	429.7924	403.6919	0.0085
2-005 EXIT ION	48.7358	644.4661	0.357	-0.397	1.406	-28.975	0.035	-1.018	770.7924	720.5457	0.0177
-3-011 GRND REF	0.0000	733.6080	0.405	-0.349	-3.299	28.173	-0.000	1.000	872.6850	822.4383	0.0177
0+000 ENTR ION	52.9741	830.3362	0.446	-0.308	-0.104	29.043	-0.277	1.223	983.3256	933.0789	0.0177
0+000 MIN DIST	189.8217	1107.5272	0.514	0.454	4.581	0.000	0.000	-1.771	1312.8736	1240.1070	0.0259

THIS RAY CALCULATION TOOK 8.113 SEC

ELEVATION ANGLE OF TRANSMISSION = 45.000000 DEG

	HEIGHT KM	RANGE KM	AZIMUTH DEVIATION XMTR DEG	AZIMUTH DEVIATION LOCAL DEG	ELEVATION XMTR DEG	ELEVATION LOCAL DEG	POLARIZATION REAL	POLARIZATION IMAG	GROUP PATH KM	PHASE PATH KM	ABSORPTION DB
0+000 XMTR	0.0000	0.0000				45.000	-0.000	1.000	0.0000	0.0000	0.0000
0+000 ENTR ION	52.9759	52.3246	-0.000	-0.000	45.000	45.471	-0.113	1.100	74.6128	74.6128	0.0000
-3-011 RCVR	200.0000	200.2014	-0.132	0.607	43.622	28.480	-0.000	2.293	295.1649	278.4398	0.0058
2-006 APOGEE	209.6834	235.8629	-0.040	-1.746	40.109	3.110	0.000	-1.831	352.1649	306.3131	0.0098
3-006 WAVE REV	209.4360	241.0969	0.021	-1.982	39.431	-0.266	0.000	-1.623	360.1649	310.1892	0.0103
3-011 RCVR	200.0000	274.3788	0.389	-1.415	34.427	-23.456	0.000	-1.142	410.8543	336.8372	0.0137
6-005 EXIT ION	38.9028	444.9524	0.574	-0.027	2.978	-44.472	0.004	-1.000	659.8543	566.5130	0.0206
0+000 GRND REF	0.0000	484.7060	0.576	-0.025	-2.180	44.114	-0.000	1.000	715.5626	622.2214	0.0206
0+000 ENTR ION	52.9777	538.6621	0.576	-0.022	3.168	44.599	-0.105	1.094	791.3392	697.9980	0.0206
-3-011 RCVR	200.0000	691.5163	0.548	0.796	12.769	26.953	-0.000	2.159	1015.3617	905.3752	0.0266

THIS RAY CALCULATION TOOK 9.813 SEC

X01 TEST CASE 11/05/74

CHAPX WAVE DIPOLY EXPZ2 APPLETON-HARTREE FORMULA EXTRAORDINARY WITH COLLISIONS

FREQUENCY = 6.000000 MHZ, AZIMUTH ANGLE OF TRANSMISSION = 45.000000 DEG

ELEVATION ANGLE OF TRANSMISSION = 60.000000 DEG

		HEIGHT KM	RANGE KM	AZIMUTH DEVIATION XMTR DEG	AZIMUTH DEVIATION LOCAL DEG	ELEVATION XMTR DEG	ELEVATION LOCAL DEG	POLARIZATION REAL	POLARIZATION IMAG	GROUP PATH KM	PHASE PATH KM	ABSORPTION DB
0+000	XMTR	0.0000	0.0000				60.000	-0.000	1.000	0.0000	0.0000	0.0000
0+000	ENTR ION	52.9786	30.2936	-0.000	-0.000	60.000	60.272	-0.053	1.049	61.0908	61.0908	0.0000
0+000	RCVR	200.0000	114.4182	-0.157	-0.851	59.325	50.578	-0.000	1.250	239.2552	225.6343	0.0050
-1-004	APOGEE	225.8382	146.6930	-0.354	5.402	55.872	-5.298	0.000	-2.002	324.2552	253.2681	0.0114
-1-004	WAVE REV	225.8382	146.6930	-0.354	5.402	55.872	-5.298	0.000	-2.002	324.2552	253.2681	0.0114
0+000	RCVR	200.0000	168.1639	-1.383	16.876	+8.744	-62.499	0.000	-1.025	395.1225	273.3158	0.0173
-2-005	EXIT ION	26.7751	230.9196	-6.447	13.564	5.561	-69.411	0.000	-1.000	588.1225	453.6124	0.0223
0+000	GRND REF	0.0000	240.7118	-7.009	13.003	-1.083	69.320	-0.000	1.000	616.7330	482.2229	0.0223
-3-011	ENTR ION	52.9836	260.0662	-7.992	12.021	10.299	69.499	-0.026	1.025	673.3376	538.8275	0.0223
0+000	RCVR	200.0000	313.9111	-10.115	8.555	30.682	63.564	-0.000	1.699	837.7013	691.0808	0.0271

THIS RAY CALCULATION TOOK .7.518 SEC

ELEVATION ANGLE OF TRANSMISSION = 75.000000 DEG

		HEIGHT KM	RANGE KM	AZIMUTH DEVIATION XMTR DEG	AZIMUTH DEVIATION LOCAL DEG	ELEVATION XMTR DEG	ELEVATION LOCAL DEG	POLARIZATION REAL	POLARIZATION IMAG	GROUP PATH KM	PHASE PATH KM	ABSORPTION DB
0+000	XMTR	0.0000	0.0000				75.000	-0.000	1.000	0.0000	0.0000	0.0000
0+000	ENTR ION	52.9846	14.0759	-0.000	-0.000	75.000	75.127	-0.024	1.023	54.8374	54.8374	0.0000
0+000	RCVR	200.0000	52.6875	-0.302	1.334	74.783	71.357	-0.000	1.087	213.9181	202.1284	0.0046
1-004	APOGEE	230.9183	67.5504	-1.511	-8.853	73.111	8.237	0.000	-3.876	314.9181	220.4673	0.0134
9-005	WAVE REV	230.2533	70.7691	-0.322	-11.181	72.307	-4.474	0.000	-1.950	330.9181	221.5782	0.0147
0+000	RCVR	200.0000	106.6297	5.511	-13.543	61.084	-46.475	0.000	-1.024	434.9673	249.2747	0.0232
-1-006	EXIT ION	29.3420	220.9908	12.392	-6.856	6.551	-55.885	0.000	-1.000	651.9673	450.5608	0.0292
0+000	GRND REF	0.0000	240.7589	13.557	-6.291	-1.083	55.706	-0.000	1.000	687.4459	486.0386	0.0292
0+000	ENTR ION	52.9836	276.3403	14.370	-5.479	9.565	56.027	-0.069	1.064	751.4563	550.0489	0.0292
-3-011	RCVR	200.0000	375.3507	15.794	-3.644	25.987	45.376	-0.000	1.374	937.9577	722.8911	0.0343

THIS RAY CALCULATION TOOK 9.049 SEC

X01 TEST CASE 11/05/74

CHAPX WAVE DIPOLY EXPZZ APPLETON-HARTREE FORMULA EXTRAORDINARY WITH COLLISIONS

FREQUENCY = 6.000000 MHZ, AZIMUTH ANGLE OF TRANSMISSION = 45.000000 DEG

ELEVATION ANGLE OF TRANSMISSION = 90.000000 DEG

		HEIGHT KM	RANGE KM	AZIMUTH DEVIATION XMTR DEG	AZIMUTH DEVIATION LOCAL DEG	ELEVATION XMTR DEG	ELEVATION LOCAL DEG	POLARIZATION REAL	POLARIZATION IMAG	GROUP PATH KM	PHASE PATH KM	ABSORPTION DB
0+000	XMTR	0.0000	0.0000				90.000	-0.000	1.000	0.0000	0.0000	0.0000
-3-011	ENTR ION	52.9908	0.0000			90.000	90.000	-0.000	1.009	52.9908	52.9908	0.0000
-3-011	MAX LAT	54.9908	0.0000			90.000	90.000	-0.000	1.012	54.9908	54.9908	0.0000
-6-011	WAVE REV	69.2408	0.0000			90.000	90.000	-0.002	1.019	69.2408	69.2408	0.0000
-3-009	WAVE REV	119.7365	0.0003	-145.262	0.004	90.000	90.000	-0.000	1.018	119.7408	119.7331	0.0018
2-007	WAVE REV	175.1532	0.1157	-145.266	-180.000	89.961	89.993	-0.000	1.021	176.7408	173.9276	0.0029
0+000	RCVR	200.0000	0.3028	-145.266	-180.000	89.911	88.779	-0.000	1.030	207.1658	194.9665	0.0047
4-006	WAVE REV	230.2036	0.9818	-145.266	0.000	89.747	89.207	-0.000	1.052	262.1658	213.5202	0.0089
-3-003	WAVE REV	238.2305	3.6109	-145.266	0.000	89.099	-11.160	-0.000	4.505	310.1658	215.7522	0.0127
-2-003	APOGEE	238.0027	4.0933	-145.266	-0.000	88.978	-33.291	0.000	-5.686	318.1658	215.8346	0.0134
-6-011	RCVR	200.0000	15.3973	-145.266	0.000	85.450	-75.213	0.000	-1.080	411.0570	238.1693	0.0285
2-005	EXIT ION	25.6746	48.6937	-145.266	0.000	27.535	-78.963	0.001	-1.000	596.0570	410.5365	0.0252
-3-011	GRND REF	0.0000	53.7019	-145.266	0.000	-0.242	78.918	-0.000	1.000	622.2176	436.6971	0.0252
-3-011	ENTR ION	52.9956	63.9944	-145.266	0.000	39.224	79.011	-0.003	1.003	676.2116	490.6911	0.0252
0+000	RCVR	200.0000	92.7180	-145.266	0.000	64.368	77.233	-0.000	1.006	833.2790	635.5670	0.0300

THIS RAY CALCULATION TOOK 13.161 SEC

Appendix 8d. Listing of Punched Card Output (ray sets) for Sample Case

```
X01  TEST CASE
 CHAPX      6.500+000 3.000+002 6.200+001 5.000-001 0.000+000 0.000+000 0.000+000
 WAVE       2.500+002 1.000+002 1.000-001 0.000+000 1.000+002 1.000+002 0.000+000
DIPOLY      8.000-001 0.000+000 0.000+000 0.000+000 0.000+000 0.000+000 0.000+000
 EXPZ2      3.650+004 1.000+002 1.480-001 3.000+001 1.400+002 1.830-002 0.000+000
X01X         0 40000255000  2000000     60000   4500000          0          0 -1003T
  1581469 14911561      2   -29      0 1514389  4513 -1610     11      0     0 -1721M
  1581469 14911561      2   -29      0 1514389  4513 -1610     11      0     0 -1722M
  1581469 29000482     -0    10    738 2875068 80425 68182     22      0     0 -1003G
  1579016 43050842     -7    76     -0 4278561109482 91246     33      0    -0  9323M
X01X         0 40000255000  2000000     60000   4500000   1500000          0 -1003T
  1721392  6041034     15   184      0  635731  8122 -3264      8      0     0 -1481M
  1721392  6041034     15   184      0  635731  8122 -3264      8      0     0 -1482M
  1721418 12129251     46   -23  14656 1211094 81100 58053     17      0    -0  1003G
  1719566 18286204     59   -65     -0 1854769 92641 58123     26      0     0 -1953M
X01X         0 40000255000  2000000     60000   4500000   3000000         -0  1003T
  1915641  3549408    -23   219      0  407964 17828 -7003      8      0     0 -1531M
  1915641  3549408    -23   219      0  407964 17828 -7003      8      0     0 -1532M
  1916346  7336080    405  -349  28173  733203139482 89236     18      0    -0  1003G
  1898217 11075272    514   454      0 113843017444410167 7     26      0     0 -1773M
X01X         0 40000255000  2000000     60000   4500000   4500000         -0  1003T
        0  2002014   -132   607  28480  285194  9971 -6754      6      0    -0  2291R
  2096843  2743788    389 -1415 -23456  342980 67874 -6143     14      0     0 -1142R
  2096843  4847060    576   -25  44114 484589230974137632     21      0    -0  1003G
        0  6915163    548   796  26953 729880285982175495     27      0    -0  2163R
X01X         0 40000255000  2000000     60000   4500000   6000000         -0  1003T
        0  1144182   -157  -851  50578  231305  7950 -5671      5      0    -0  1251R
  2258382  1681639  -1383 16876 -62499 262993132130 10323     17      0     0 -1032R
  2258382  2407118  -7009 13003  69320 240697376036241525     22      0    -0  1003G
        0  3139111 -10115  8555  63564 376316461386314765     27      0    -0  1103R
X01X         0 40000255000  2000000     60000   4500000   7500000         -0  1003T
        0   526875   -302  1334  71357  207034  6884 -4906      5      0    -0  1091R
  2309183  1066297   5511 -13543 -46475 227435207533 21840     23      0     0 -1022R
  2309183  2407589  13557 -6291  55706 240745446701245294     29      0    -0  1003G
        0  3753507  15794 -3644  45376 430430507528292462     34      0    -0  1373R
X01X         0 40000255000  2000000     60000   4500000   9000000         -0  1003T
        0     3028214734180000 88779  200000  7166 -5034      5      0    -0  1031R
  2382305   153973214734      0-75213 200610210447 37559     20      0     0 -1082R
  2382305   537019214734      0 78918  53702568516382995     25      0    -0  1003G
        0   927180214734      0 77233 221057612222414510     30      0    -0  1013R
```

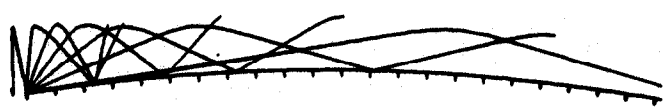
The first card is the title card.
The second card contains the name of the electron density model plus parameters W101-W107.
The third card contains the name of the perturbation model plus parameters W151-W157.
The fourth card contains the name of the magnetic field model plus parameters W201-W207.
The fifth card contains the name of the collision frequency model plus parameters W251-W257.

For description of remaining cards, see figures 1 and 2.

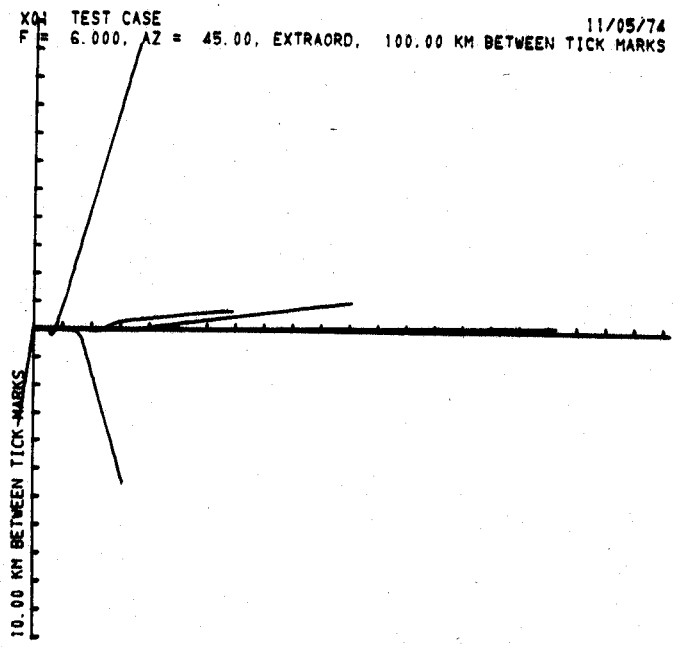
Appendix 8e. Ray Path Plots for Sample Case

Projection of raypath on vertical plane

X01 TEST CASE 11/05/74
F = 6.000, AZ = 45.00, EXTRAORD, 100.00 KM BETWEEN TICK MARKS



Projection of raypath on ground for sample case



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7. AUTHOR(S) R. Michael Jones and Judith J. Stephenson		9. Project/Task/Work Unit No.
8. PERFORMING ORGANIZATION NAME AND ADDRESS U. S. Department of Commerce, Office of Telecommunications, Institute for Telecommunication Sciences, Boulder, Colorado 80302		10. Contract/Grant No.
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15. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)*

This report describes an accurate, versatile FORTRAN computer program for tracing rays through an anisotropic medium whose index of refraction varies continuously in three dimensions. Although developed to calculate the propagation of radio waves in the ionosphere, the program can be easily modified to do other types of ray tracing because of its organization into subroutine.

The program can represent the refractive index by either the Appleton-Hartree or the Sen-Wyller formula, and has several ionospheric models for electron density perturbations to the electron density (irregularities), the earth's magnetic field and electron collision frequency.

For each path, the program can calculate group path length, phase path length, absorption, Doppler shift due to a time-varying ionosphere, and geometrical path length. In addition to printing these parameters and the direction of the wave normal at various points along the ray path, the program can plot the projection of the ray path on any vertical plane or on the ground and punch the main characteristics of each ray path on cards.

The documentation includes equations, flow charts, program listings with comments, definitions of program variables, deck set-ups, description of input and output, and a sample case.

KEY WORDS:
Appleton-Hartree formula; computer program; ionosphere; radio waves; ray tracing; Sen-Wyller formula; three-dimensional.

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