Appendix F

Procedures to recover energetic proton number fluxes from the Omnidirectional detector responses.

Figure F.1 shows the omnidirectional detector configuration. The detector itself is a right cylinder, 0.3 cm high with front and back surface areas of 0.5 cm^2. The diameter of the detector is 0.798 cm, the circumference of the cylinder is 2.507 cm, and the area of the cylinder side is 0.752 cm^2. The 60º reference angle refers to the half-angle subtended by the spherical shell absorbers in detectors P6 and P7.

Energetic particles may enter the front face of the solid-state detector at an angle ϑ to the central axis of the detector system and, should the Tungsten absorber surrounding the detector permit, a normal angle γ to the side of the cylinder. Particles energetic enough to penetrate the Tungsten shielding, and incident at a ϑ angle greater than 90º, may enter the solid-state detector through the back face.

For a given angular distribution of protons incident upon the detector system, the number of particles passing into the detector, which is the detector count rate, may be computed. In this way a relationship, unique to that angular distribution, can be established to relate the detector count rate to the directional number flux of protons within specific energy ranges.

The fundamental equation that relates the directional number flux of particles within a specified energy range, $J(\Phi,\vartheta)$ incident from a polar angle ϑ and an azimuthal angle Φ to the number flux through an elemental area dA on the surface of a detector is

Equation F.1 Number flux = $J(\Phi,\vartheta)\, \sin\vartheta \cos\vartheta \, d\vartheta \, d\Phi \, dA$

The integration of this equation over all angles Φ and ϑ and the entire surface of the detector yields the sensor count rate due to particles within the specified energy range.

The procedure to accomplish this for two specific proton angular distributions follows.

The first case is for a totally isotropic particle distribution. That is, the directional flux of protons in units of particles cm^{-2} sec^{-1} $ster^{-1}$ is constant from all directions. This proton angular distribution might be encountered at the magnetic equator in the inner radiation belt, but it is not likely to be encountered at the rather low altitude of the POES satellites.

The second case treated is for a proton angular distribution that is constant over all angles ϑ up to 105º but has zero intensity at ϑ between 105º and 180º. This proton angular distribution might be encountered by the NOAA/POES satellite at polar latitudes during energetic solar particle events. At these locations the central axis of the omnidirectional detectors is nearly parallel to the magnetic field. During such events the solar protons are often isotropic over the upper hemisphere and those protons incident downwards at ϑ angles between 75º and 90º are magnetically mirrored at altitudes below the satellite and appear as upward going protons at ϑ angles between 90º and 105º.

The analysis in both cases begins with the count rates registered by the P6, P7, P8, and P9 detectors, with the count rate determined from the telemetered count value divided by the accumulation period. These values are CR_6 CR_7, CR_8 and CR_9. The analysis is designed to return directional number fluxes within 4 energy ranges, J_6(16-35 MeV), J_7(35-70 MeV), J_8(70-140 MeV) and J_9(140-500 MeV).

Case 1. Completely Isotropic Angular Distribution

Step 1 is the calculation of the P9 sensor count rate from protons over the energy range 140 to 500 MeV. There are three parts to this calculation. The first is the contribution from protons entering the front face of the detector, the second from protons entering the side of the detector, and the third from particles entering the back face of the detector.

For a directional number flux J_9 the number of particles entering the front face of the detector is given by

Equation F.2 $$N_1 = \iiint J_9 \, sin\vartheta \, cos\vartheta \, d\vartheta \, d\Phi \, dA$$

Because J_9 is independent of ϑ and Φ, it may be moved outside the integral. Particles of energies greater than 140 MeV incident from all ϑ angles between 0º and 90º and from all Φ angles from 0º to 360º, reach the detector so that the integration of angle yields

Equation F.3 $$N_1 = \pi J_9 \int dA$$

The integration over the area of the detector is 0.5 cm^2 so the final contribution to the sensor count rate from particles entering the front face of the detector is

Equation F.4 $N_1 = 0.5 \pi J_9$.

Since the particle population is assumed completely isotropic, the number of particles entering the detector through the back face is the same as through the front and their contribution to N_3 is

Equation F.5 $N_3 = 0.5 \pi J_9$.

Similarly, the contribution from particles entering the detector side is

Equation F.6 $N_2 = .752 \pi J_9$, and

the total number of particles entering the detector $N_1 + N_2 + N_3$, which is the count rate, is,

Equation F.7 $CR_9 = 1.752 \pi J_9 = 5.504 J_9$.

The geometric factor for the P9 detector for the case of a totally isotropic proton distribution is 5.504 cm^2 ster.

The directional number flux of protons in the energy range 140 to 500 MeV may then be calculated from the P9 sensor count rate CR_9 from

Equation F.8 $\mathbf{J_9 = 0.182\ CR_9}$ (J_9 in units of particles $cm^{-2}\ sec^{-1}\ ster^{-1}$)

Step 2 is the calculation of the P8 sensor count rate from protons over the energy range 70 to 500 MeV.

The design of the P8 omnidirectional detector is similar to the P9 in that the detector shielding is the same in all directions. Using the same procedure outlined for P9, the count rate in detector P8 due to an isotropic flux of protons over the energy range 70 to 500 MeV is

Equation F.9 $CR_8 = 5.504(J_8 + J_9)$.

The directional number flux of protons in the energy range 70 to 140 MeV may then be calculated from the P8 and P9 sensor count rates, CR_8 and CR_9 from

Equation F.10 $\mathbf{J_8 = 0.182(CR_8 - CR_9)}$ (units of particles $cm^{-2}\ sec^{-1}\ ster^{-1}$)

Step *3* is the calculation of the P7 sensor count rate from protons over the energy range 35 to 500 MeV.

The P7 detector design allows protons of energies greater than 70 MeV access to the solid-state detector from all directions, but access of protons between 35 and 70 MeV is permitted only through the hemispherical shell absorber which subtends approximately a 60º half-angle as viewed from the center of the detector. The P7 sensor count rate will have contributions, first from protons between 70 and 500 MeV entering from all directions, and second from protons between 35 and 70 MeV which enter through the dome. The latter contribution is calculated from the fundamental equation above, where the limits on the Φ integration remain 0º to 360º but the ϑ integration is now 0º to 60º. Protons entering through the dome have no access to either the sides or the back face of the detector, so using the procedure outlined above

Equation F.11 $CR_7 = 0.375\,\pi J_7 + 5.504(J_8 + J_9)$ or

Equation F.12 $CR_7 = 1.178\,J_7 + 5.504(J_8 + J_9)$.

The directional number flux of protons in the energy range 35 to 70 MeV may then be calculated from the P7 and P8 sensor count rates, CR_7 and CR_8 , from

Equation F.13 $\mathbf{J_7 = 0.849(CR_7 - CR_8)}$ (units of particles $cm^{-2}\ sec^{-1}\ ster^{-1}$)

Step 4 is the calculation of the P6 sensor count rate from protons over the energy range 16 to 500 MeV.

The design of the P6 omnidirectional detector is similar to the P7, except for the thickness and material used in the dome detector. Following the procedure outlined in step 3, the calculated P6 count rate is

Equation F.14 $CR_6 = 0.375\,\pi\,J_6 + 0.375\,\pi\,J_7 + 5.504(J_8 + J_9)$ or

Equation F.15 $CR_6 = 0.375\,\pi\,J_6 + CR_7$.

The directional number flux of protons in the energy range 16 to 35 MeV may then be calculated from the P6 and P7 sensor count rates, CR_6 and CR_7, from

Equation F.16 $\mathbf{J_6 = 0.849(CR_6 - CR_7)}$ (units of particles $cm^{-2}\ sec^{-1}\ ster^{-1}$)

Case 2. An Angular Distribution Isotropic to 105º Polar Angle and Empty Between 105º and 180º

A procedure similar to Case 1 is followed for computing particles entering the detectors through the front face of the detector. The ϑ integration for the back face of the detector is carried out only from 90º to 105º. The contribution from particles entering the side of the detector is obtained by using a full azimuthal (0º to 360º) integration for polar angles, γ, between 0º and 15º, but only a one-half azimuthal (0º to 180º) integration for polar angles between 15º and 90º to reflect the absence of particles arriving from below at angles greater than 105º.

Step 1*.* Following the earlier case,

Equation F.17 $N_1 = 0.5\,\pi J_9$

Equation F.18 $N_2 = 0.033\,\pi J_9$ reflecting the integration of ϑ from 90º to 105º, and

Equation F.19 $N_3 = 0.752\,\pi(0.067+0.467)\,J_9$

the first term from the 0º to 15º integration and the second the 15º to 90º integration over the angle γ.

The sum $N_1 + N_2 + N_3$, which is the count rate, is then determined by:

Equation F.20 $CR_9 = 0.934\,\pi J_9 = 2.935\,J_9$

The directional number flux of protons in the energy range 140 to 500 MeV may then be calculated from the P9 sensor count rate, CR_9, from

Equation F.21 $\mathbf{J_9 = 0.341\,CR_9}$ (units of particles $cm^{-2}\,sec^{-1}\,ster^{-1}$)

Step 2, following the above procedure, yields the directional number flux of protons in the energy range 70 to 140 MeV calculated from the P8 and P9 sensor count rates, CR_8 and CR_9, as

Equation F.22 $\mathbf{J_8 = 0.341(CR_8 - CR_9)}$ (units of particles $cm^{-2}\,sec^{-1}\,ster^{-1}$)

The expressions for J_6 and J_7 in case 2 are the same as in case 1 because the protons between 15 and 70 MeV are isotropic over the 60º angle subtended by the hemispherical shell absorber.

Equation F.23 $\mathbf{J_7 = 0.849(CR_7 - CR_8)}$ (units of particles $cm^{-2}\,sec^{-1}\,ster^{-1}$)

Equation F.24 $\mathbf{J_6 = 0.849(CR_6 - CR_7)}$ (units of particles $cm^{-2}\,sec^{-1}\,ster^{-1}$)

Similar procedures may be followed to obtain the conversion between sensor count rates and proton directional number fluxes in the four energy ranges for more complicated particle angular distributions.

It should be noted that these procedures neglect the increased shielding provided by the satellite structure and material (circuit boards, brackets, etc.) in the MEPED unit, that for some arrival angles, may increase the proton energy needed to reach the detector.